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INVESTIGATIONS OF THE CORONA AT THE SUMATRA ECLIPSE OF JANUARY 14, 1926

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ABSTRACT

Photometric measures of the brightness of the solar corona at the eclipse of January 14, 1926, were made both visually and photographically at Benkoelen, Sumatra.

Visual measurements of the intensity of the *total light of the corona and surrounding sky* were made with a *Macbeth illuminometer* loaned by the U.S. Bureau of Standards. The resultant light-curve places the *minimum illumination* received from the *corona and sky* as 0.138 foot-candles (0.0128 meter-candles) and the time of occurrence at $7^h35^m50^s.2$ G.C.T.

Photometric registrations were also obtained photographically on both ordinary and isochromatic plates with a *coronal photometer* designed by Professor E. S. King. The resultant plates, when measured with the thermo-electric photometer, yield for the *brightness of the corona and sky* within a 3° circle: *photographic magnitude*, -11.38; *photovisual magnitude*, -12.00; and *color index*, $+0.62$. Corresponding values determined by King for the 1925 eclipse are: -10.96 photographic, -11.61 photovisual, and $+0.65$ color index. Measures of these plates indicate a *coronal light 40 per cent greater* at the *1926 eclipse* than at the *eclipse of 1925* showing an *apparent increase with the sun-spot numbers*.

Radiometric measures of the water-cell transmission of the coronal radiation were attempted with a 20-inch reflecting telescope and vacuum thermocouple, by use of the method and program carried out at Middletown in 1925. High humidity and a thin veil of cirrus cloud, however, interfered seriously with the observations entailing a large amount of infra-red absorption which explains the failure to obtain measurable indications of coronal radiation by means of the thermopile.

The *ultra-violet light of the corona* was registered by the use of a silvered quartz lens.

The eclipse of January 24, 1925, afforded unusual opportunity for photometric and radiometric measures of the corona at the time of totality in which several Harvard observers could share.[1]

The eclipse of January 14, 1926, following in less than a year with nearly twice the duration of totality made it seem especially desir-

[1] E. S. King, *Popular Astronomy*, **33**, 289, 1925; Stetson and Coblentz, *Astrophysical Journal*, **62**, 128, 1925.

able to carry out, so far as possible, investigations of identical problems, not only as a check on the methods employed, but also to detect any appreciable change in the nature or activity of the corona with the changing sun-spot cycle.

If any systematic change in emission from the corona is apparent with the changing solar cycle, it is obvious that many observations, standardized so far as possible as to methods, must be made at frequent intervals before any concomitant variation may be definitely established.

The three American expeditions to observe the 1926 eclipse represented Swarthmore College, the United States Naval Observatory, and Harvard University, all of which were located in Sumatra. Detailed descriptions of the more general circumstances connected with the eclipse, the three expeditions, and the selection of sites have already appeared elsewhere.[1] It is perhaps sufficient to say that in choosing sites and programs for eclipse work, co-operation with the work of other observers is most essential so that there may be no unnecessary duplication of effort, especially in the same locality. Our party, which was located on the west coast at Benkoelen, attempted no direct photography of the corona as that was being well cared for by the Swarthmore expedition with several cameras devoted to that end. The Harvard expedition, therefore, devoted itself chiefly to the measurement of the intensity and distribution of coronal radiation which, for convenience in discussion, may be divided into four lines of attack: (1) photometric measurements with a visual illuminometer of the total light of the corona and a determination of the light-curve during minimum; (2) photographic photometry of the intensity of the coronal radiation in blue and in yellow light on an absolute scale, thus giving a direct comparison of the intensity of the coronal light of the eclipse of 1926 with that of 1925, and, at the same time, a determination of color index making a similar intercomparison again possible; (3) the attempted measurements of the heat of infra-red radiation of the corona by means of a vacuum thermocouple and filtering screens in connection with a 20-inch reflecting telescope; (4) photographic registration of coronal light in ultra-vio-

[1] John A. Miller, *Popular Astronomy*, **34**, 349, 1926; F. B. Littell, *ibid.*, p. 438, 1926; H. T. Stetson, *ibid.*, p. 413, 1926.

let radiation only, by means of a quartz lens heavily silvered, through which no visual light could penetrate, as shown by careful tests.

PHOTOMETRIC MEASUREMENTS

For the measurement of the total light of the corona visually a Macbeth illuminometer manufactured by the Leeds and Northrup Company was selected as a suitable and most convenient instrument for the purpose. One of these instruments, which belonged to the National Bureau of Standards and which was carefully calibrated before and after trans-shipment, was generously loaned us by the Bureau for the Sumatra eclipse. The instrument is essentially a very compact form of photometer. The comparison standard is a small incandescent lamp attached to a sliding bar calibrated to read illumination directly in foot-candles. The lamp was carefully calibrated and fed from dry cells through a rheostat at a fixed amperage consumption. The sliding bar is adjusted by means of a rack-and-pinion movement and is calibrated to read from 1 to 25 foot-candles. By means of suitable absorbing screens placed either on the side containing the comparison lamp or on the side receiving light from the object under observation (the test side), intensities of illumination varying from 0.003 to 10,000 foot-candles and upward can be measured.

The instrument is so simple in operation and so completely standardized that it commends itself as an ideally portable instrument for the visual photometry of eclipses. The rapidity with which the settings can be made and the small probable error of the measures involved make it especially desirable for obtaining photmetric data for the intercomparison of the brightness of the corona at successive eclipses. The compactness of the entire outfit together with its extreme portability make it of special value where expeditions to remote regions are involved.

Preliminary checks of the calibration were made by Coblentz. Many measurements of the intensity of normal illumination from the sun near the meridian and at the predicted time of eclipse and of the full moon were consistently made by Coblentz, Stetson, and Arnold as shown in Table I. Constant practice for the week preceding totality was maintained by Arnold who made the final observations

TABLE I

NORMAL ILLUMINATION BY THE SUN AND THE MOON AT BENKOELEN, SUMATRA, AND AT WASHINGTON, D.C.

Sumatra	Normal Illumination, Foot-Candles	Observer	Remarks
	Sun		
Mid-Java time:			
1925			
Dec. 26, 11:00 A.M. . .	10,800	W.W.C.	Sky clear
Dec. 26, 11:00 A.M. . .	260	W.W.C.	Sky illumination 30° W. of sun
Dec. 26, 11:20 A.M. . .	11,900	W.W.C.	Sky perfectly clear
Dec. 26, 2:00 P.M. . .	11,100	W.W.C.	Sky clear
Dec. 27, 11:45 A.M. . .	8,900	W.W.C.	Very faint haze
Dec. 28, 2:40 P.M. . .	9,200	W.W.C.	Clear
Dec. 30, 2:15 P.M. . .	10,300	H.T.S. and W.W.C.	Very clear
1926			
Jan. 3, 3:30 P.M. . .	10,200	W.A.	Thin passing clouds
Jan. 11, 1:30 P.M. . .	11,000	W.W.C.	Faint clouds
Jan. 11, 1:40 P.M. . .	10,300	W.W.C.	Faint wisps of clouds
Jan. 11, 2:58 P.M. . .	8,800	W.A. and W.W.C.	Hazy around the sun
Jan. 14, 1:30 P.M. . .	10,050	W.A.	Beginning of eclipse
Jan. 14, 2:55 P.M. . .	0,138	W.A.	Totality
Jan. 16, 1:45 P.M. . .	10,400	W.W.C.	Very faint, barely visible clouds near sun
Washington East. Std. Time:			
Apr. 2, 11:53 A.M. . .	10,300	W.W.C.	Temp., 46° F.; clouds on horizon
Apr. 9, 11:45 A.M. . .	10,800	E.G.A. and W.W.C.	Sky clear and free from dust after a rain
	Moon		
Sumatra:			
1925			
Dec. 25, 11:55 P.M. . .	0.0083	W.W.C.	
Dec. 31, 5:30 A.M. . .	.0175	W.W.C.	Full moon; alt., 30°; thin clouds
1926			
Jan. 1, 9:50 P.M. . .	.0089	H.T.S. and W.W.C.	2 days past full; alt., 25°
Jan. 1, 10:15 P.M. . .	.0122	W.A. and W.W.C.	Thin clouds
Jan. 28, 8:45 P.M. . .	.0272	W.A. and W.W.C.	Full moon; alt., 45°; sky perfectly clear; position at sea in Malacca Straits; Singapore to Penang
Washington:			
Apr. 25, 8:30 P.M. . .	.0175	W.W.C.	
Apr. 25, 9:35 P.M. . .	.0180	W.W.C.	Sky clear; temp., 47° F.
Apr. 26, 9:25 P.M. . .	.0183	W.W.C.	1 day before full moon; temp., 46° F. Sky becomes overcast at 9:30
Apr. 27, 9:00 to 12:00 P.M.	00	W.W.C.	Full moon; cloudy
Apr. 28, 10:00 P.M. . .	.0120	W.W.C.	Temp., 48° F.; clouds on horizon
Apr. 28, 11:00 P.M. . .	.0178	W.W.C.	Clear
Apr. 28, 11:30 P.M. . .	.0180	W.W.C.	Clear
May 25, 8:45 P.M. . .	0.0086	W.W.C.	Near full moon; alt., 15°; sky clear

throughout the eclipse. The illuminometer is illustrated in Figure 1, and the light-curve plotted logarithmically from observations throughout the eclipse is shown in Figure 2, giving the minimum intensity at maximum eclipse as 0.138 foot-candles (0.0128 meter-candles).

Since the return of the instrument to the Bureau of Standards the illuminometer has again been tested for calibration and there can

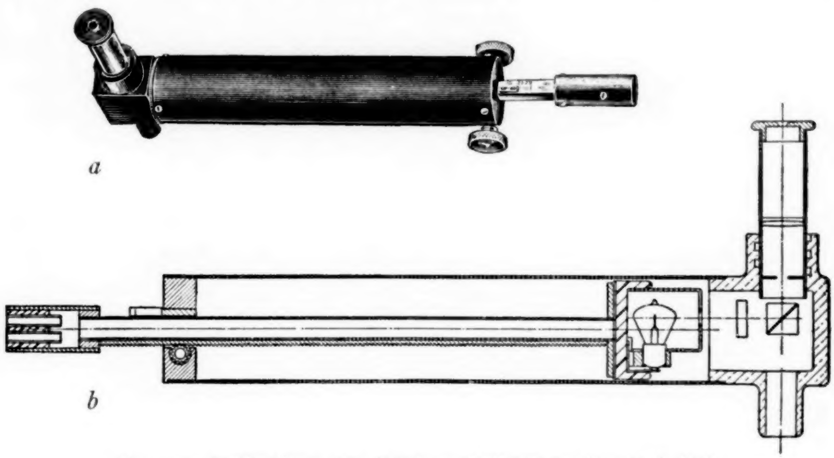

FIG. 1.—*a*) Illuminometer, Leeds and Northrup ($\frac{1}{8}$ actual size)
b) Cross-section of illuminometer

be little uncertainty of the value of the normal illumination observed.

In view of published values of coronal brightness for previous eclipses[1] the value of 0.138 foot-candles seems surprisingly large. It should be remarked, however, that the elbow attachment with diffusing screen which was used in front of the illuminometer and directed toward the object measured received light of practically the entire sky during totality.

The form of the light-curve near minimum is plotted on an enlarged scale in Figure 3 and gives for the mean value of the time for mid-totality, when the chronometer correction is applied, G.C.T.

[1] S. A. Mitchell, *Eclipses of the Sun*, p. 345, 1923; *Astrophysical Journal*, **49**, 137, 1919; *ibid.*, **60**, 280, 1924; *Publications of the Astronomical Society of the Pacific*, **37**, 85, 1925; *Astrophysical Journal*, **62**, 202, 1925; *Harvard College Circular*, No. 286, 1926; *Transactions of the Illuminating Engineering Society*, **20**, 565, 1925.

$7^h\,35^m\,50^s.2$. It is of interest to compare this with the predicted time for our camp at Benkoelen which was G.C.T. $7^h 35^m 50^s.5$.

PHOTOGRAPHIC PHOTOMETRY

As an independent method of measuring the integrated brightness of the solar corona, a coronal photographic photometer devised

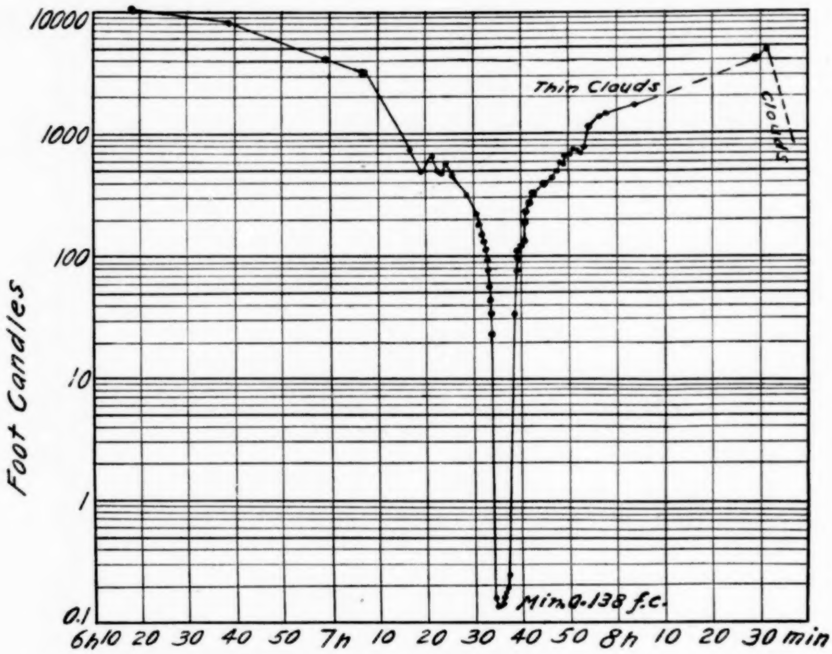

FIG. 2.—Total normal illumination during solar eclipse January 14, 1926. (Measurements with Macbeth illuminometer.)

by Professor E. S. King was employed for simultaneous exposures through suitable screens upon ordinary and isochromatic plates, thus affording data for the color index of the corona as well as the intensity of its illumination.

This form of photometer has already been fully described by Professor King in the report of the observation of the eclipse of 1925[1] by the Harvard College Observatory, and has afforded valuable data for the intercomparison of the two eclipses. Of the four photometers described by Professor King, that designated A was taken to Su-

[1] E. S. King, *Harvard College Circular*, No. 286, 1926.

matra, together with one Hefner lamp and amyl acetate (*c. p.*) for imposing the standard exposures for purposes of comparison.

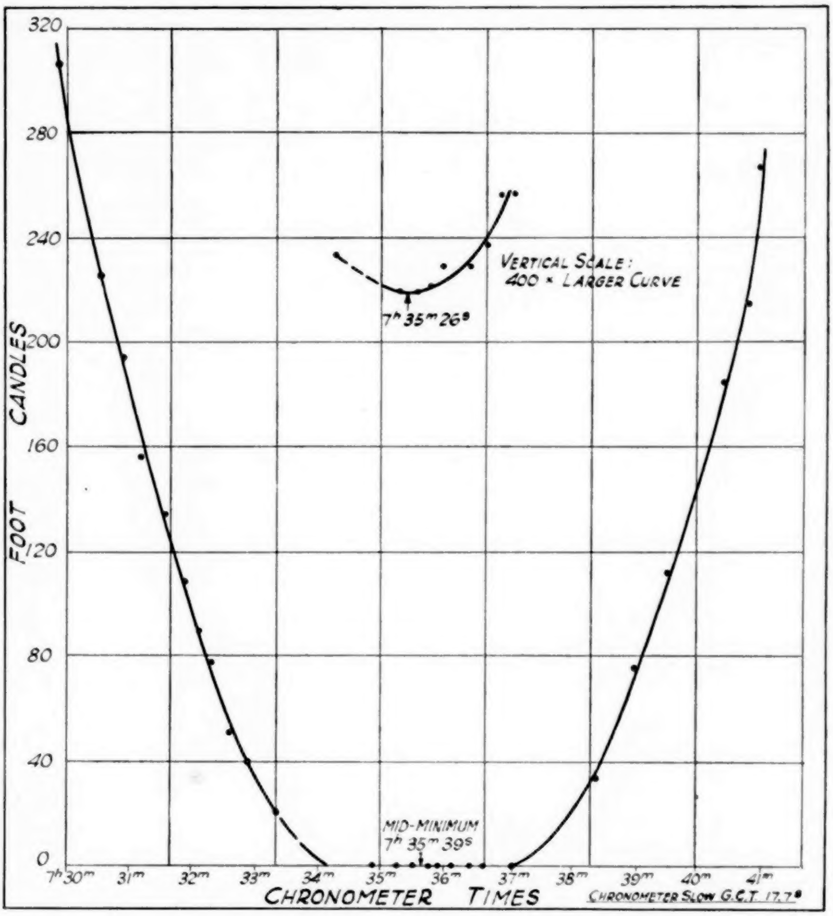

FIG. 3.—Illumination curve through totality

The absorption in magnitudes of the neutral and yellow screens placed in front of the ordinary and isochromatic plates, respectively, given by Professor King are as follows:

PHOTOMETER A

	Light	Dark	Diff.
Blue........	1.90	3.80	1.90
Yellow......	1.72	2.86	1.14

Four plates were exposed with the photometer directed to receive normal illumination from the corona. Each "plate" consisted of one strip of Cramer high-speed emulsion placed in juxtaposition with a similar strip of Cramer instantaneous isochromatic emulsion. By means of the shade glasses each plate received light of three gradations, the first strip receiving light without the yellow filter, whereas the second strip (the "iso") received light of the same gradations but traversing the yellow filter which was intense enough to exclude all the blue.

Two exposures to the corona were made upon each plate, one of four seconds and one of two seconds, the plate-holder being shifted slightly between exposures. The aperture employed for Plates I and IV covered 3° and for Plates II and III, 6° of the sky. The eclipse exposures were made by Spurr.

The exposures for standardization were all made on the evening after the eclipse by Stetson and Arnold. Following the procedure of Professor King, the strips of plate in each side of the plate-holders were turned end for end before the standardized comparison exposures were made. In this way the comparison exposures would fall on the opposite side of each strip from the eclipse records. The plate-holders were replaced in the photometer which was directed to the Hefner lamp and exposures were made with the lamp first at 1 m and then at a distance of 1.58 m from the front of the plate, the diminution of intensity in the second position being exactly 1 mag.

Great care was taken to shade the lamp and dark room in such manner as to minimize sources of error from stray reflections, and exposures were made only when the standard flame was burning steadily, unaffected by drafts. The standardizing exposures were all made with the full aperture of the photometer, the time for Plates I and III being four seconds and for Plates II and IV, four seconds each.

The plates were developed on the following day in paramidophenol (1 part to 16 of water) for five minutes at a temperature between 65° and 70° F. (20° C.) maintained by the use of an external cooling bath regulated by ice. The plates were developed four strips at a time in one tray and fixed, washed, and dried under identical conditions.

On account of the possible deteriorating effect of the tropical cli-

mate the plates were kept in tin receptacles at Benkoelen and check plates unexposed were developed prior to eclipse day to detect any deterioration. In this test the isochromatic plates, which it was feared would be the first to suffer, showed on development at standard temperature less fog than was found on some Eastman tropical (isochromatic) plates locally purchased, sealed in tin from the factory.

In addition to the four plates exposed to the corona, two plates were exposed to the sky at the end of the exposure of the fourth plate by shifting the photometer from the corona to a point 8° east of the sun. The first of these plates receiving four and two second exposures through half (3°) aperture and the second four and two second exposures through full (6°) aperture. These plates were standardized and developed according to the procedure described for the coronal plates.

The data of all the exposures on each plate including the standardizations are summarized as follows:

Plate	Designation	Aperture on Corona	Exposures		
Order of Exposure	Plate No.		Corona	Hefner Full Aperture at 1 M	Hefner Full Aperture at 1.58 M
1	III, III*a*	3°	4^s, 2^s	4^s	4^s
2	IV, IV*a*	6	4, 2	2	2
3	V, V*a*	6	4, 2	4	4
4	VI, VI*a*	3	4, 2	2	2
			Sky 8° from sun		
5	I, I*a*	3	4, 2	4	4
6	II, II*a*	6	4, 2	2	2

The first column gives the number of the plate in the order of exposure. The second gives the identifying mark placed on each plate just after its standardization and before development. The unmodified number is for the strip of ordinary or "blue" plate in the correspondingly numbered pocket of the plate-holder; the suffix *a* designates the corresponding strip of isochromatic plate or "yellow" plate in the same compartment. The reason that the identifying marks do not correspond with the numbers as given in column 1 is that the order of exposure was purposely changed just before totality

in order that plates designated I, I*a*, which possibly had been fogged (through an accidental slipping of the plate-holder slide), should not be used on the corona, but rather on the sky which was relatively of

TABLE II

REDUCED MEAN MEASURES OF PHOTOMETRIC PLATES

GALVANOMETER DEFLECTIONS—δ

BLUE

Light-Values in Mag. Diffs.	Plate Numbers							
	III		IV		V		VI	
	Stand.	Corona	Stand.	Corona	Stand.	Corona	Stand.	Corona
0.00	0.921	0.948	0.706	0.787	0.833	0.833	0.899	0.929
1.00	.820	.861	.584	.773	.765	.846	.735	.906
1.74	.749	.827	.451	.697	.696	.755	.722	.815
2.74	.525	.637	.194	.598	.502	.691	.284	.778
3.50	.304	.378	.047	.372	.341	.450	.329	.458
4.50	0.064	0.124	0.000	0.196	0.125	0.225	0.000	0.363

YELLOW

Light-Values in Mag. Diffs.	Plate Numbers							
	III*a*		IV*a*		V*a*		VI*a*	
	Stand.	Corona	Stand.	Corona	Stand.	Corona	Stand.	Corona
0.00	0.932	0.755	0.688	0.656	0.892	0.827	0.930	0.729
1.00	.892	.384	.643	.523	.871	.742	.841	......
1.68	.865	.346	.502	.245	.822	.358	.828	.295
2.68	.578	.100	.216	.067	.593	.133	.318	.336
2.64	.639	.098	.200	.079	.591	.126	.595	......
3.64	0.234	0.000	0.041	0.000	0.216	0.072	0.104	0.102

SKY-BRIGHTNESS

Blue					Yellow				
Light-Values in Mag. Diffs.	Plate Numbers				Light-Values in Mag. Diffs.	Plate Numbers			
	I		II			I*a*		II*a*	
	Stand.	Sky	Stand.	Sky		Stand.	Sky	Stand.	Sky
0.00	0.882	0.257	0.597	0.519	0.00	0.898	......	0.607	......
1.00	.795	0.118	.456	.418	1.00	.787	......	.517	......
1.74	.631	......	.369	.164	1.68	.742	......	.413	......
2.74	.445	......	.162	.083	2.68	.499	......	.120	......
3.50	.202	......	.040	.014	2.64	.452	......	.148	......
4.50	0.024	......	0.018	0.016	3.64	0.146	......	0.065	......

less importance. Subsequent development showed a slight fogging of the lower end of the suspected plates but there was no apparent interference with the photometric record.

The plates have all been measured on the thermo-electric photometer described elsewhere,[1] and reduced in accordance with the formula $\delta = (D - D')/D$, where D is the galvanometer deflection representing the transparency through the unexposed portion of the plate and D', the galvanometer deflection representing the transparency through the "images" or exposed portions.

The reduced mean measures for the several plates together with the light-values of the exposures, first, for the standardizing tests, and second, for the coronal exposures, are given in Table II. From the tabulated values, curves were drawn showing the relationship between galvanometer deflections and exposure values in magnitude differences.

The curves for the test exposures for Plates III and IV and the corresponding curves for the coronal exposures are shown in Figure 4. The full-line curves are for the standardizing exposures to the Hefner lamp and the dotted curves are for the exposures to the coronal light. The average differences in abscissae between the dotted curve and the corresponding full curves represent the measured magnitude differences between the corona and the Hefner lamp. Similar curves were plotted for the other plates, and the results of all the plates are gathered into Table III.

The magnitudes of the Hefner lamp, photographic and photovisual, are taken from values published by King.[2]

That the photographic and photovisual values for the illumination from the 6° circle differ from the corresponding values for the 3° circle by 0.2 mag., but with opposite sign, probably is not of material significance, though the apparent increase in the color index may possibly indicate a reddening in the adjacent sky near mid-totality. The fact that both the 6° exposures were taken nearer mid-totality than were the 3° exposures may have some bearing on the apparent discrepancy.

[1] Stetson, *Astrophysical Journal*, **43**, 253, 1916; Stetson and Carpenter, *ibid.*, **58**, 36, 1923.

[2] *Harvard Annals*, **80**, 120, 1916; *Harvard College Circular*, No. 286, 6, 1926.

As the sky was not ideally clear for photometric purposes, the variation in these values may not be greater than the experimental error involved.

A glance at Table III shows far more accordant values for the 3° plates than for the 6° plates. It seems not impossible that the thin

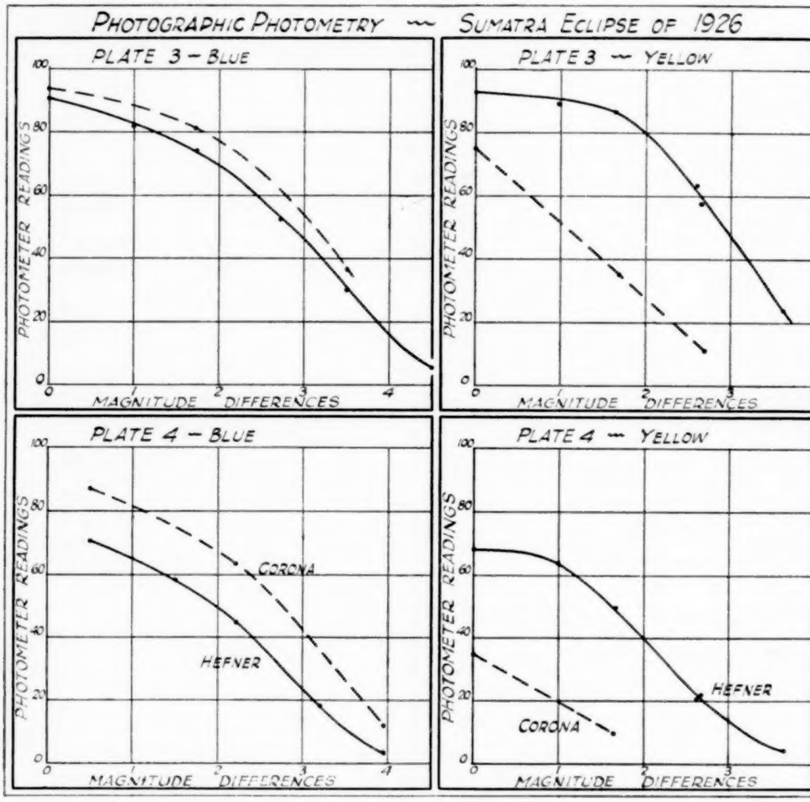

FIG. 4

clouds in the neighborhood of the corona are directly responsible for the large range of magnitude differences found in both the four-second and two-second exposures made on Plate III. The mean of these measured values for these two four-second exposures was taken for single values entered in the table for Plate VI.

If one were to assume that the bulk of the light of the corona is

included in the 3° circle, a reasonable assumption, we find the mean of the very consistent values for the 3° plates to give for the photographic magnitudes for the corona and superimposed sky in the 1926 eclipse to be −11.38 mag., and the photovisual magnitude to be −12.00, giving a color index of 0.62. The corresponding values found

TABLE III

BRIGHTNESS OF CORONAL LIGHT—SUMATRA, 1926
Magnitude Differences Are Hefner Lamp—Corona

Plate No.	Aperture	Blue Mag. Diff.			Yellow Mag. Diff.		
		4 Sec.	2 Sec.	Mean	4 Sec.	2 Sec.	Mean
III	3°	−0.70	− 0.74	−0.72	+1.90	+ 1.90	+1.90
IV	6	(.19)	.86		1.32	1.62	1.47
V	6	.49	.44	.48	1.63	2.14	1.89
VI	3	−0.68		−0.68	+1.89		+1.89
Mean for 3°			0.70			+ 1.89	
Mag. of Hefner		Ptg.	10.68		Ptv.	−13.89	
Mag. of corona, 3° circle			11.38			−12.00	Color Index, +0.62
Mean for 6°			0.67			+ 1.76	
Mag. of Hefner			10.68			−13.89	
Mag. of corona, 6° circle			11.35			12.13	.78
King's values, 1925	3		10.96			11.61	.65
	6		−11.40			11.71	+0.31
1926–1925	3		+ 0.42 brighter			0.39 brighter	
	6		+ 0.05 fainter			− 0.42 brighter	

PHOTOGRAPHIC BRIGHTNESS OF THE SKY

Plate No.	Aperture	Mag. Blue
1	3°	−8.09
2	6	−9.92

by King from the results of Harvard observers in 1925 were: photographic magnitude −10.96, photovisual −11.61, giving a color index of 0.65 for the corona and superimposed sky within a circle 3° in diameter. This would indicate an increase in brightness of 0.42 in the photographic magnitude and 0.39 in the photovisual magnitude of the 1926 as compared with the 1925 eclipse.

The lower values obtained on the 6° plates as compared with the 3° plates makes inapplicable the method of King in determining values for sky brightness within the 3° circle. However, the results of the measures of the plates exposed to a region of the sky 8° from the sun during totality indicate a photographic magnitude of −8.09 for

a 3° circle and −9.92 for a 6° circle in the same region. The yellow light in this instance was too feeble to register.

The photographic magnitude of the full moon at mean distance has been given by King as $-11^{m}.20$.[1] Using this value, the photographic brightness of the 1926 corona thus determined was $0^{m}.18$ brighter than that of full moon. Corresponding values indicate the 1925 eclipse $0^{m}.24$ fainter than that of the full moon. This corresponds to an increase in brightness of about 40 per cent.

COMPARISON OF VISUAL AND PHOTOGRAPHIC METHODS

The difference in the results of the integrated brightness of the coronal light measured with the illuminometer and the photographic photometer demands further consideration.

Taking the brightness of the Hefner lamp at 1 m to be 0.0836 foot-candles (0.0078 meter-candles) and $-10^{m}.68$ as the photographic magnitude of the corona, the coronal brightness as photographed becomes 0.16 foot-candles (0.0149 meter-candles). Taking the photovisual magnitude of the Hefner lamp as $-13^{m}.89$, and of the corona as −12.00, we have a corresponding photovisual brightness of the corona of 0.015 foot-candles (0.0014 meter-candles), whereas the value obtained visually with the Macbeth illuminometer was 0.14 foot-candles (0.013 meter-candles), or nearly ten times as great.

From the table of photometric measures made with the illuminometer it will be observed that the brightness of the full moon following that of the eclipse was measured on the return trip in the Malacca Straits and with perfect atmospheric conditions. A value of 0.027 foot-candles (0.0025 meter-candles) was obtained. Correcting for the moon's distance on the date in question we obtain a value of 0.029 foot-candles (0.0027 meter-candles) for the light of full moon at mean distance (parallax = $57'.04$). The ratio of the brightness of coronal light (photovisually obtained) to the brightness of the full moon thus becomes 0.52 in general agreement with earlier values.

The ratio of the total illumination during totality to full moonlight as measured with the illuminometer, however, is about 5.1. The large number of readings on the light-curve during the eclipse leaves little doubt as to the relative accuracy of the settings and we can see

[1] *Harvard Annals*, **59**, 94, 1912.

no source of systematic error. The excess of total illumination as measured with the illuminometer as compared with the light-intensity of the corona measured photographically is probably to be chiefly accounted for on the basis of the fact that the diffusing screen used in front of the illuminometer received light from the entire sky.

Although the coronal light may be of the same order of brightness as full moonlight, it must be remembered that a considerable amount of diffused light overspreads the entire sky due to the sun's rays falling on atmospheric particles just outside the shadow cone, the sky during eclipse being much less dark than the sky surrounding a full moon. In the latter case what sky illumination exists comes from the full moonlight alone.

For purposes of measuring total illumination during totality, the illuminometer fitted with elbow diffusing screen appears to be a satisfactory and most convenient instrument. For measuring the brightness of the corona alone, we would recommend the removal of the external diffusing screen or a restriction of its field of illumination to 6°. In the latter case the instrument should be equatorially mounted or otherwise kept accurately directed to the eclipsed sun as a small difference in orientation without the diffusing-screen results in large differences in values of the intensities measured.

It is suggested that this may be one of the chief sources of the discrepancies recorded in measurements of coronal intensities at the New England eclipse of 1925 as recorded in the *Transactions of the Illuminating Engineering Society*.[1] It is patent to remark that these observers when using the horizontal diffusing test plate obtained values for the illumination during totality of from 0.15 to 0.45 foot-candles.

RADIOMETRIC MEASURES

The experience of our radiometric measures at Middletown at the eclipse of January 24, 1925, made it seem particularly desirable to provide a larger mirror for the radiometer. We accordingly adapted to the purpose a 20-inch (50 cm) mirror from the Harvard Observatory. The exceedingly short focus (112 cm) was particularly suitable for radiometric work. The reflector was mounted equatorially with the thermocouple and accessories attached to an eyepiece

[1] *Transactions of the Illuminating Engineering Society*, **20**, 565, 1925.

at the side of the tube. As was the case at Middletown, the thermopile was so constructed and so mounted that both junctions could be placed simultaneously on the dark of the moon for a zero reading and then by a slight lateral displacement the couple could be shifted so that either receiver could be set on the inner corona while the cold junction remained on the moon's disk. Requisite slides provided for the introduction of water cell or a glass screen immediately in front of the thermopile. Focusing was quickly accomplished by the usual rack-and-pinion movement. The thermopile container was made transparent front and back, so that the observer could see the receiver silhouetted against the coronal light and make the several settings without movement of the telescope mounting other than that communicated by the driving clock.

Although the amount of radiation from earth shine coming from the eclipsing moon is at a maximum it is comparatively insignificant. On the other hand, in the event of a veil of clouds (as was experienced at Benkoelen), the radiation from the earth's atmosphere to the thermocouple leaves a poor substitute for a zero of radiation as was so nearly ideally reached in the settings at Middletown, where a perfect sky and arctic temperature allowed neither haze nor much, if any, water vapor.

A grant from the Rumford Committee of the American Academy of Arts and Sciences made possible the use of a d'Arsonval galvanometer specially constructed to meet our special requirements by the Leeds and Northrup Company. It had a sensitivity of 8 mm per microvolt, a total resistance of 12 ohms, a period (i.e., time to attain a single swing) of four seconds, and an external critical damping resistance of 7 ohms as determined by the usual methods.

It is relevant to add that the constants of d'Arsonval galvanometers are determined by the instantaneous application of the full voltage. The moving coil, acting like a motor, generates a counter e.m.f. which in turn tends to stop the rotation of the coil in a certain time, depending upon the external resistance. On the other hand, when a thermopile, of the type used, is exposed to radiation, it generates about 90–95 per cent of the maximum voltage in the first second. As a consequence, the movement of the galvanometer coil is slower, the counter e.m.f. is different, and the external critical damp-

ing resistance is somewhat larger than required by the ordinary laboratory tests. In the present instance, an external resistance of about 16 ohms was required to dampen critically the coil and attain a maximum deflection in four seconds when activated by the thermopile. Of the 16 ohms required for critically damping the movement of the coil 6 ohms were supplied by the thermopile.

The galvanometer scale was at a distance of 2 m. It was illuminated by six 3.5 volt lamps joined in parallel. Two dry cells supplied the necessary current and gave an efficient and convenient means of scale illumination.

The thermopiles were essentially the same as described in previous papers,[1] except that the receivers were smaller. They consisted of four elements of bismuth-silver with receivers of pure tin $1 \times 1 \times 0.018$ mm and specially connected to fit the problem. The bismuth wire was 0.08 mm in diameter pressed flat. Opinions may rightfully differ on the question of the area of the corona to be intercepted by the thermopile. The receivers used in the present instance were 4 mm long and 1 mm wide and were purposely constructed of these relatively large dimensions in order to integrate a considerable part of the radiation from the corona.

Two transmission screens were provided as part of the equipment for analyzing the component radiation of the corona. One was a specially selected glass screen (0.16 mm in thickness), and well adapted for absorbing the infra-red rays from 8 μ to 15 μ which may be transmitted by the earth's atmosphere. The second screen provided was a water cell similar to that used in the radiometric measurements at Middletown in 1925. The water cell was constructed of a block of glass $4 \times 4 \times 1$ cm with a central hole 2 cm in diameter. The windows were of crystalline quartz about 2 mm in thickness. The quartz windows together with a filling aperture 7 mm in diameter, sufficiently large for filling purposes, left little difficulty in the way of keeping the cell clean.

The spectral transmission of 1-cm layer of water has been repeatedly determined by various observers during the past three decades, and all are in agreement in observing a transmission of about 20 per cent at 1.3 μ and complete opacity at 1.4 μ. Hence it would

[1] *Astrophysical Journal*, **62**, 128, 1925.

appear wise to adhere to the value of 1.4 μ as the logical limitation of the spectrum instead of 1.3 μ as sometimes used. For a radiator at 2500°–3000° C. and having its maximum emission near 1 μ (or even for the solar spectrum), a transmission of 18–20 per cent at 1.3 μ is a very perceptible amount of the total.

The reflecting mirror already described was silvered and coated with highly diluted Egyptian lacquer immediately before shipping and sealed in tin for the transportation. A few days before the eclipse, the lacquer was easily removed by means of amyl acetate (simply by pouring on the amyl acetate repeatedly and draining the surface), and the silvering was found in perfect condition. Experience thus shows that this is an excellent method of preserving silvered surfaces during shipment to remote stations.

Since laboratory experiments at the Bureau of Standards have shown that ultra-violet light deteriorates a lacquered silver mirror more rapidly than an uncovered silvered surface,[1] we were careful not to expose the mirror in full sunlight for a great length of time prior to the eclipse.

Atmospheric conditions at Benkoelen were not as bad as might have been expected during the rainy season. The usual experience was to have cloudiness in the morning and forenoon, then a clearing off more or less complete until the middle of the afternoon, thus leaving the sky clear at the time of day the eclipse would occur. Sometimes the sequence was shifted earlier in the day so that the sky would become clouded at the expected time of solar eclipse. On the eventful day it became cloudy at noon. A thick cloud overcast the southwest quadrant of the sky, but moving rapidly southward, it left the sky practically clear in the region of the incoming solar rays about a half-hour before totality. This was followed by a bank of faint diaphanous cirrus clouds drifting southward just before and during totality. A motion-picture film of the progress of the eclipse taken by Wilson Powell of the Swarthmore party 100 yards distant has verified our impressions of the state of sky thus described.

After the eclipse the sky seemed entirely clear for perhaps a half-hour or more, when clouds again gathered and it became overcast for the rest of the day.

[1] Coblentz, *U.S. Bureau of Standards Paper*, No. 342, 1919.

Conflicting reports as to weather conditions sent out by the American and British observers are easily reconciled as the British camp at Benkoelen was in Fort Marlborough nearly a mile to the northward and apparently at time of totality clear of the cloud shadows.

While this faint haziness and the thin diaphanous clouds did not interfere seriously with the photographic work, this, together with high absolute humidity, could explain our failure to obtain strong positive indications of radiation from the corona. We were situated the farthest south of any of the stations at Benkoelen, and hence the last to be clear from the southward-moving haze and thin cirrus clouds, which from our experience at Lick and the Lowell observatories greatly interfere with the radiometric measurements. As this haziness lessened the sharpness of contrast between the dark image of the moon's disk and the white light of the corona, the infra-red radiation from these finely divided particles might have easily offset and obscured the radiation from the corona itself.

The thermocouple and all electrical circuits were tested both before and after totality by the exposure of one junction to sky illumination, leaving us with the conclusion that the only explanation for the negligible deflections received during totality must have been the lack of sufficient radiant energy received through the humid atmosphere.

Meteorological observations made by Mr. Arnold during the progress of the eclipse gave the following data:

Time	Thermometer		Barometer	Wind
	Wet Bulb	Dry Bulb		
G.C.T. 6^h46^m	78°.4	88°.1	29.905	Nil
56	78.0	86.2	.903	....
7 06	78.1	85.6	.898	....
16	76.0	83.8	.900	....
35	76.0*	82.0*	Mid-totality	....
8 03	76.1	82.5	.890	....
14	76.1	83.6	.888	....
26	78.0	85.6	.880	....
35	77.2	86.1	.878	....
43	79.3	86.8	29.880	....

* By estimation.

From the wet- and dry-bulb readings it appears that the relative humidity at Benkoelen at the time of totality was 76 per cent, making the water content at 82° F. (27.8° C.) 8.84 grains per cubic foot or approximately 20.25 gm per cubic meter.

From the considerations of Humphreys[1] on the extent of water-vapor in the upper atmosphere, it would appear that an approximation to the total water content of the atmosphere for an average day could best be obtained by the application of Hann's value $d = 2.3e$, where d equals the depth in millimeters of a horizontal layer of water equivalent to all the water-vapor overhead, and e equals the vapor pressure in millimeters of mercury at the place of the observation.

From Smithsonian meteorological tables we find the vapor pressure at 82° F. (27.8° C.) and 76 per cent humidity to be 21.1 mm. This gives, then, for d, 48.5 mm of water. As Hann's value is based chiefly on atmospheric soundings in middle latitudes and as humidity in tropical regions extends to considerably higher altitudes, it seems reasonable to suppose that the foregoing value for d should be increased from 10 to 15 per cent. A conservative estimate therefore for the depth of a horizontal layer of liquid water equivalent to all the water-vapor overhead at Benkoelen would appear to be 5–6 cm. This is entirely aside from any consideration of absorption or scattering of radiation by the thin cirrus cloud overspreading the sun.

In contrast to the conditions at Benkoelen in 1926 we find for the eclipse of January 24, 1925, quite the opposite conditions. The official United States Weather Bureau report at Hartford, Connecticut, at $9^h\ 15^m$ E.S.T. (the time of the eclipse), records the following data: thermometer, dry bulb +0°.2 F. (−17°.7 C.), wet bulb −1°.0 F. (−18°.3 C.); barometer, 30.57 in. (776 mm). The conditions at Middletown, 18 miles (30 km) south where observations of coronal radiation were being made, could not have been very different.

We find, then, from these data a relative humidity of 60 per cent, giving a water content at 0° F. of 0.317 gm per cubic foot or 0.742 gm per cubic meter. The corresponding vapor pressure is 0.58 mm, and by Hann's formula the depth of a layer of liquid water equivalent to all the water-vapor above Middletown (Lat. 41°33′) on the day in question was 1.33 mm. This is only about one-eighth the amount of

[1] W. J. Humphreys, *Bulletin of Mt. Weather Observatory*, 4, 121, 1911.

precipitable water presupposed by Pettit and Nicholson in a recent discussion of radiometric measurements of coronal radiation.[1] It shows, however, what astoundingly small interference was introduced at Middletown by atmospheric water-vapor, whereas at Benkoelen we had the equivalent of a sheet of precipitable water thirty-six times as thick as that at Middletown.

If we write for the intensity of transmitted radiation $\log I_t/I = k\ e^{-t}$ for Middletown and $I'_t/I = k\ e^{-36t}$ for Benkoelen, we have for the ratio of the intensity of coronal radiation transmitted through the Sumatran atmosphere to that transmitted at Middletown $I'_t/I_t = e^{-35t}$.

Whether we assume that 60 per cent of coronal radiation is absorbed by 1 cm of water as was indicated by the measures of Stetson and Coblentz[2] or adopt the smaller value of 30 per cent later favored by Pettit and Nicholson,[3] the results are substantially the same in demonstrating that inasmuch as absorption increases exponentially, we shall find the amount transmitted by the atmospheric water-vapor at Benkoelen to be such a small fraction of the observable radiation on January 24, 1925, at Middletown as to make deflections on the galvanometer vanishingly small, even in spite of our larger outfit of about ten times the previous sensitivity.

In rehearsal of the eclipse program at Benkoelen the day before the event, an eclipse model was set up with a disk of black cardboard as the eclipsing moon and a wax candle serving as a source of "coronal" radiation near the moon's limb. To produce the required angular diameter, the whole affair was placed at a distance of 175 feet (53 m) from the radiometer. Measures of the radiation from the candle with one junction of the thermocouple set on the image of the candle in the telescope while the other junction was superimposed upon the image of the black disk were made repeatedly, alternating the two junctions involved. The mean deflection of the galvanometer for the candle radiation as compared to radiation from the black disk was 3.0 mm.

With a quantity of 5 cm of precipitable water between the instrument and the corona on eclipse day, it is evident that a sufficient

[1] *Astrophysical Journal*, **64**, 136, 1926.

[2] *Loc. cit.*

[3] *Astrophysical Journal*, **62**, 202, 1925.

amount of coronal energy could not have reached the thermocouple to produce reliably readable deflections. For any further observations in tropical regions a far more sensitive outfit would be needed.

It is needless to point out further the exceptionally favorable conditions at the eclipse of 1925 for the purpose of radiometric measurements of the corona.

There remains only to mention the photography of the corona by means of a silvered quartz lens previously used by Professor King in 1925.[1] The quartz lens and camera were attached to the tube of the reflecting telescope carrying the radiometric outfit, and exposures were made on Cramer high-speed plates, while the radiometric apparatus was being operated. In spite of some slight movement of the apparatus due to necessary and unavoidable manipulation of the radiometric outfit, images of the corona in ultra-violet light were secured, giving some evidence of the distribution of the inner coronal material emitting these high-frequency radiations. The fact that the impressions were due entirely to wave-lengths in the invisible spectrum was verified through the courtesy of Dr. Davidson of the English party who photographed for us the solar spectrum through the heavily silvered lens by means of the quartz spectrograph of the British expedition. The transmission band of the silvered lens was from 3080 to 3400 A.

Whatever the nature of the inner corona the plate bears testimony to the unmistakable evidence of the emission of radiation of very short wave-lengths of sufficient intensity to record an image of the corona for a radial distance from the sun's limb of about 1′ of arc in the case of an exposure of about fifteen seconds and of 6′ of arc from the exposure of one hundred and ninety seconds, the duration of totality.

SUMMARY

From our observations and experience at the Sumatra eclipse it seems worth while to call attention to a few conclusions which we may well include at this place. First, the matter of the photometry of the corona is probably a more complicated affair than may at first be supposed. Conflicting values are, in many instances, quite considerably due to conditions experienced at different eclipses and not

[1] *Harvard College Circular*, No. 286, 9, 1926.

necessarily due to observational errors. The question of total sky illumination will vary with the season and locality and furthermore with the duration of the eclipse. All of these conditions clearly affected the scattered light of the atmosphere lying within the penumbra. It is, therefore, highly desirable that methods of coronal photometry be standardized as well as possible, in order that results from eclipse to eclipse may be comparable.

It should be pointed out, however, that, granting atmospheric conditions and methods of photometry to be the same, it seems hardly likely that differences in coronal brightness as measured from year to year can be taken without modification, as representing quantitatively actual changes in the intrinsic brightness of the corona itself. Whatever law may be taken for varying brightness with increasing distance from the solar limb, all will agree that the brightness is an inverse function of the distance from the sun. The brightest part of the corona is therefore unquestionably the innermost part of the corona which will be more exposed at eclipses of short duration, and less exposed at eclipses of long duration. The duration of the eclipse, therefore, must be an important factor upon the measured brightness of coronal light. If observations are made either visually or photographically, it seems important that as many records be made as possible during the total phase, in order that the actual minimum of the illumination curve may be determined. By continued measurements of coronal brightness at subsequent eclipses of varying duration, it may be possible to determine how much of the change in illumination is due to the duration of the eclipse itself and how much to actual change in coronal activity. Only when such corrections have been applied can we really get a quantitative measurement of coronal intensity which may be compared with the sun-spot cycle or other solar activity by tracing the radiation effects of the corona itself. One can hardly overemphasize, therefore, the importance of coronal photometry at every eclipse so that records may be furnished as soon as possible for making true comparisons of coronal activity feasible. When this has been accomplished and methods of observation standardized, perhaps many conflicting measurements of coronal brightness throughout the history of eclipses will appear in large measure reconciled.

Concerning radiometric measures it appears from our experience that tropical conditions with high humidity render thermo-electric measures of coronal radiation exceedingly difficult and even if made with sufficiently sensitive apparatus, it is doubtful if the test of water-cell absorption could yield much of significance unless made at high altitudes or under vastly improved meteorological conditions compared with those obtained at Benkoelen, Sumatra, in 1926.

Considering the very great gamble involved in eclipse work in the tropics especially in the rainy season, we may feel fortunate for as favorable conditions for general observation as were afforded, and it is hoped that the experience and photometric results at least will prove of some value in linking together the altogether too few measures of coronal light.

The authors wish to express their very great appreciation to the Dutch government, to the Resident, and to Secretary Oberman at Benkoelen, for the free transportation of our party and goods when on Dutch soil and for their hospitality, and the many courtesies shown us there. We shall not soon forget the generous good will of Professor John A. Miller in allowing us to share many of the facilities of the Swarthmore expedition and a portion of their camp site while in Benkoelen.

We are indebted to the director of the Bureau of Standards for the loan of the illuminometer and for the leave of absence granted Dr. Coblentz, and to Dr. A. Hamilton Rice of the American Geographical Society for the loan of Mr. Arnold. For the loan and preparation of some of the apparatus we are especially grateful to Professor King and the director of the Harvard College Observatory.

With grateful appreciation we acknowledge the financial assistance of Mrs. Robert Wheeler Willson and Mr. R. Bruce Barbour which helped make the expedition possible.

Acknowledgment is due Mr. L. B. Andrews, assistant in astronomy at Harvard University, for assistance in the reduction of some of the photometric observations.

HARVARD UNIVERSITY
STUDENTS' ASTRONOMICAL LABORATORY
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INVESTIGATIONS ON PROPER MOTION[1]

TWELFTH PAPER: THE PROPER MOTIONS AND INTERNAL MOTIONS OF MESSIER 2, 13, AND 56

By ADRIAAN VAN MAANEN

ABSTRACT

Proper motions and internal motions of globular clusters.—Three pairs of photographs of M 13 and two each of M 56 and 2 have been measured with the stereocomparator in order to determine the proper motions and internal motions of these clusters. Part of the plates were taken at the 80-foot focus, part at the 25-foot focus of the 60-inch reflector; the interval between the old and new exposures ranges from nine to fifteen years.

The probable errors of an individual μ_α or μ_δ for the 80-foot- and 25-foot-focus plates is found to be 0\".030 and 0\".064, respectively, divided by the interval in years.

The absolute proper motions of the clusters have been derived by correcting the comparison stars for parallactic motion. The results are:

M 13..........	$\mu_\alpha = +0\rlap{.}''0005$	$\mu_\delta = +0\rlap{.}''0008$
M 56..........	$\mu_\alpha = -0.0013$	$\mu_\delta = +0.0066$
M 2...........	$\mu_\alpha = +0.0082$	$\mu_\delta = +0.0026$

The internal motions were analyzed into components parallel to and at right angles to the "galactic planes" of the clusters. The dispersion of the motions in these two directions is sensibly the same, which indicates that there is no pronounced rotational motion parallel to the galactic planes.

The internal motions were further analyzed into radial and tangential components. There is a small preponderance of outward motion, but it is too small to permit any definite conclusions.

The tangential component was found to be exactly zero for the 80-foot-focus plates and 0\".003 in the direction N-W-S-E for the 25-foot-focus plates. The tangential component found for the spiral nebulae was in the mean 0\".014 and 0\".018[5] for the 80-foot- and 25-foot-focus plates, respectively, and is therefore of a different order of magnitude.

No correlation was found between the tangential components and the magnitudes of the stars, although the range in the case of the 25-foot-focus plates was more than 6 magnitudes.

In *Mount Wilson Contribution*, No. 284, it was shown from photographs taken at the 80-foot focus of the 60-inch reflector that the total motion of the globular cluster M 13, as well as its internal motion, is extremely small, of the order of a few thousandths of a second of arc per year only. As a check on the displacements found in spiral nebulae, which amount to a few hundredths of a second of arc per year, it was deemed worth while to investigate a few more globular clusters, especially at the 25-foot focus at which most of the spirals were photographed, as soon as material for the derivation of the proper motions should be available.

[1] *Contributions from the Mount Wilson Observatory, Carnegie Institution of Washington*, No. 338.

The present paper discusses the results from two pairs of plates of M 56 and one pair of M 2, taken at the 80-foot focus of the 60-inch reflector, and from one pair of plates each of M 13 and M 2, taken at the 25-foot focus of the same instrument. Further, the measures communicated in *Contribution* No. 284 have been re-reduced. The material for these plates is assembled in Table I.

TABLE I

Object	Plate Number	Exposure Time	Date	Quality	Observer
M 13	π 5*	30^m	1913, May 26	g	van Maanen
M 13	π 119†	30	1914, April 18	g	van Maanen
M 13	π 3728†	30	1923, June 8	g	van Maanen
M 13	π 3878*	$31\frac{1}{4}$	1924, May 30	fg	van Maanen
M 13	P 133§	300	1912, June 15	fg	Pease
M 13	B 242§	300	1926, July 3-5	fg	Humason
M 56	π 153*	30	1914, July 26	fg	van Maanen
M 56	π 160†	30	1914, July 27	fg	van Maanen
M 56	π 3754*	30	1923, July 7	fg	van Maanen
M 56	π 4151†	30	1925, Aug. 24	g	van Maanen
M 2	π 239*	15	1914, Sept. 14	g	van Maanen
M 2	π 4154*	20	1925, Aug. 24	g	van Maanen
M 2	P 5†	100	1911, July 2	fg	Pease
M 2	B 248†	100	1926, July 6	fg	Humason

The plates marked π were taken with the Cassegrain arrangement of the 60-inch reflector (equivalent focal length, 80 feet); those marked P or B, at the Newtonian focus (25 feet) of the same instrument. All plates were measured differentially in pairs with the stereocomparator. The grouping of these plates is indicated by the signs *, †, and §, following the plate numbers.

The usual methods of measurement and reduction were followed, the plates being measured in four positions each, with east, west, north, and south in the direction of the increasing readings of the micrometer. For the reduction, the two series of measures in right ascension, as well as those in declination, were combined into single series. The quantities so derived were then reduced to seconds of arc per year by multiplying by the factor:

$$q=\frac{\text{No. seconds of arc per mm on plate}}{\text{No. revolutions of micrometer screw per mm}\times\text{interval in years}}\,.$$

The resulting quantities, m, were used as the second members of the equations of condition (1) from which the final proper motions were derived.

$$\left.\begin{array}{l}\mu_\alpha+a+bx+cy+dx+exy+fy=m_\alpha\\ \mu_\delta+a'+b'x+c'y+d'x^2+e'xy+f'y^2=m_\delta\end{array}\right\} \qquad (1)$$

Here $a, b, c, \ldots . e', f'$, are the plate constants; x and y, the co-ordinates of the stars measured; and μ_α and μ_δ, the resulting annual motions relative to the mean motion of the comparison stars. The plate constants were derived by least-squares solutions from equations (1) for the comparison stars, and were then substituted into the equations of condition for all the stars measured.

In the case of M 13, Shapley had indicated which stars might be members of the cluster and which not. The comparison stars were selected from the latter, the selection being based on distance from the center, magnitude, and color-index. For M 2 and 56 no such data were available. Accordingly, a limiting distance was adopted outside of which it was assumed that no stars were members of the cluster. The cluster stars were chosen well within these limits. It is, however, still possible that some of the comparison stars may belong to the cluster, and that some of the cluster stars are in reality foreground or background stars; but the number in either case will probably be small and cannot disturb the mean motions very seriously.

In *Contribution* No. 284, dealing with the motion of M 13, fifty stars were used for comparison purposes. It was found that the star s had a proper motion of $0\overset{''}{.}110$ in declination, while p had a motion of $0\overset{''}{.}054$ in the same co-ordinate. It would therefore have been better had these two stars been excluded from the derivation of the plate constants. Furthermore, on the long-exposure plates taken at the 25-foot focus, stars 831 and 1019, being located in the denser part of the cluster, could not be measured; and, as it was desirable to use the same set of comparison stars for the pairs of plates taken at both the 80- and 25-foot foci, a re-reduction of the measures in *Contribution* No. 284 was made, excluding these four stars. Accordingly, for M 13, forty-six comparison stars were used, while sixty-two cluster stars were measured at the 80-foot focus,

and seventy-three at the 25-foot focus, the latter including some of the very faintest stars measurable on the plates of five-hour exposure. For these twenty-two faint stars no magnitudes are available, but they are estimated to be between the eighteenth and nineteenth magnitudes. In Table III their magnitudes are given as *ff*.

For M 56, sixty-five comparison stars were selected and fifty cluster stars. Six of the cluster stars are, however, considerably brighter (12.7–13.6 mag.) than the other cluster stars (14.4–16.3 mag.) and

TABLE II

PROBABLE ERROR OF MEASURES

Object	P.E. in μ_α	P.E. in μ_δ	Interval in Years between Plates	Focal Length in Feet
M 13	0″.0038	0″.0036	11	80
	0.0031	0.0028	9	80
	0.0050	0.0037	14	25
M 56	0.0031	0.0035	9	80
	0.0023	0.0037	11	80
M 2	0.0017	0.0023	11	80
	0.0052	0.0036	15	25

it is possible that they may be foreground stars. These stars, numbered 101, 105, 125, 131, 144, and 149, have accordingly been excluded from the derivation of the mean motion of the cluster, as well as from the discussion of the internal motions.

For M 2 the same set of thirty-eight comparison stars was used for both the 80-foot- and the 25-foot-focus plates. For the former, seventy-six cluster stars were chosen, of which three, Nos. 164, 175, and 176, were later rejected as too bright and too far from the center, while for the 25-foot-focus plates the number of cluster stars was seventy-six. Numbers 175 and 176 were also excluded from the discussion of the motion of the cluster.

In the case of M 13, twenty of the cluster stars were measured on all sets of plates; in the case of M 2, twenty-eight.

As stated before, the magnitudes for the stars of M 13 are due to Dr. Shapley, those for M 56 were derived by Dr. Pannekoek from Mount Wilson plates, while those for M 2 are by Mr. J. A. Brown, to all of whom I wish to express my sincere thanks for their kind co-operation.

TABLE III

CO-ORDINATES AND ANNUAL DISPLACEMENTS IN M 13

(The Motions Are Given in 0″.001 as a Unit)

No.	Pg. Mag.	x	y	80-Foot Focus		25-Foot Focus	
				μ_α	μ_δ	μ_α	μ_δ
Comparison Stars							
a	13.06	−9′.7	−4′.8	−5	+9	−7	+14
c	13.40	−8.2	−6.7	+5	0	+3	−3
b	14.96	−8.0	+3.0	+11	+3	+10	−4
e	14.52	−6.9	−2.4	+5	0	+2	−5
d	14.52	−6.8	+5.8	+5	−19	+8	−21
1	14.85	−6.5	+2.5	+8	−1	+6	+2
f	14.52	−6.1	−6.2	+4	0	+8	−5
10	15.14	−5.6	+1.3	+2	+3	−5	+2
13	15.50	−5.4	+2.8	+3	+2	−8	−2
15	13.06	−5.2	+5.6	−25	+19	−17	+21
19	15.91	−5.0	+3.2	+2	−4	+1	−5
36	16.13	−4.3	+1.9	−6	−7	+10	−7
g	13.30	−4.3	−6.6	−6	0	+1	−3
43	14.52	−4.2	−3.2	+2	−2	−1	+10
47	15.89	−4.0	−3.8	+2	+2	−3	+2
h	14.95	−2.9	−8.5	−6	−4	−11	+1
90	16.22	−2.8	−1.8	−18	−20	−8	+3
99	16.03	−2.6	−1.5	−2	+15	−8	−14
1120	16.17	−2.3	+0.6	−8	−13	+11	+11
234	16.16	−1.3	+3.7	+1	−2	−7	+1
i	15.00	−0.8	+8.4	0	+7	+8	+9
331	16.13	−0.8	−4.2	+6	0	−1	−2
j	15.17	−0.1	+8.0	+1	+2	−1	+1
505	16.16	0.0	+4.4	+1	+5	+4	−1
557	15.11	+0.2	+6.3	+4	−1	−3	+7
599	16.17	+0.3	+4.4	+1	−1	−11	−5
k	13.82	+0.4	−8.1	+5	−1	−3	−4
687	13.76	+0.7	−5.3	−3	0	−2	−5
762	15.95	+1.0	+4.0	+1	−1	+2	−12
814	16.20	+1.2	+2.4	+6	+1	+11	−3
831*	16.06	+1.3	−2.9	+4	−7	....	....
1015	16.14	+2.7	−4.2	+4	+2	+10	+5
1019*	12.98	+2.7	−1.4	+5	−13	....	....
l	14.34	+3.4	−6.1	+2	−1	+1	−2
1057	15.86	+3.5	−2.3	−4	+6	0	+1
1075	15.79	+4.1	−3.9	+1	+2	+10	+2
1087	13.14	+4.4	+3.9	−6	+16	−17	+4
m	14.88	+5.3	+7.6	+4	0	+5	−5
1111	16.10	+5.7	+3.0	−4	−14	+2	+11
1114	14.63	+5.8	−5.5	+3	−5	+7	+7
1115	15.71	+5.9	+6.0	+3	+6	−2	+3
n	13.90	+7.2	−3.0	+5	−1	+7	+3
*p**	13.14	+7.3	−8.0	−42	+31	−41	+36
o	14.88	+7.7	+1.2	−5	−3	−10	−5
r	14.09	+8.0	−1.0	+2	+10	+6	+2
q	14.33	+8.2	+6.2	+6	−24	−5	−18

* Rejected in deriving the plate constants.

TABLE III—*Continued*

No.	Pg. Mag.	x	y	80-Foot Focus μ_α	80-Foot Focus μ_δ	25-Foot Focus μ_α	25-Foot Focus μ_δ
Comparison Stars							
*s**	14.88	+8′.4	−6′.7	−15	−132	− 8	−135
t	15.77	+9.2	+6.5	− 2	+ 6	+10	+ 8
u	15.50	+9.3	−0.4	− 2	+11	− 1	+ 3
v	15.28	+9.8	−3.0	− 2	− 3	−12	− 6
Cluster Stars							
2	14.96	−6′.5	+2′.3	+11	− 3	− 5	+ 3
7	15.04	−6.1	+6.4	+ 5	− 2	+10	+ 4
17	14.86	−5.1	−4.6	+ 2	− 1	− 4	− 3
21	16.01	−4.9	+4.4	...	...	0	+ 5
25	15.06	−4.8	+1.8	+ 3	+ 1	+ 4	+ 1
46	15.57	−4.0	−1.9	− 9	+ 1	− 9	− 4
49*a*	*ff*	−4.0	−0.4	...	...	+14	− 2
49	14.83	−3.9	−0.2	+ 4	− 3	− 1	+ 2
51	16.63	−3.9	+0.8	...	...	+ 2	+ 2
51*a*	*ff*	−3.8	+0.7	...	...	+ 8	+ 1
54	16.20	−3.8	+2.9	...	...	− 1	− 5
59*a*	*ff*	−3.8	+3.3	...	...	+ 6	+ 1
59	15.08	−3.7	+3.2	+ 7	− 5	− 1	+ 1
70	13.45	−3.4	+2.4	− 5	+ 1	− 1	+ 2
70*a*	*ff*	−3.2	+2.6	...	...	+ 6	− 7
71	16.21	−3.3	+0.8	...	...	− 1	+ 2
71*a*	*ff*	−3.3	+0.9	...	...	+ 5	− 2
72	13.74	−3.3	−2.3	− 6	− 1	− 4	− 4
76	16.04	−3.2	−1.7	...	...	− 7	+ 1
82*a*	*ff*	−3.1	+2.7	...	...	+ 9	− 7
82	16.10	−3.0	+2.6	...	...	− 2	−13
96	13.70	−2.7	+0.2	− 3	− 1	−10	+ 2
113	16.22	−2.4	−5.0	...	...	+11	− 4
113*a*	*ff*	−2.3	−4.9	...	...	+14	− 1
118	15.03	−2.4	−0.9	+ 4	− 3	...	...
155	16.56	−2.0	−4.8	...	...	+ 7	+ 2
158	13.86	−2.0	+1.9	− 2	− 3	+ 4	−10
159	16.09	−2.0	−5.6	...	...	0	− 7
222	12.54	−1.4	−1.7	− 4	+ 1	...	...
236	16.63	−1.3	+2.8	...	...	− 8	+ 5
246	15.49	−1.2	−1.6	− 3	+ 4	...	...
252	13.84	−1.2	+2.4	+ 2	+ 1	...	...
275	16.24	−1.1	+3.9	...	...	+ 1	+ 1
275*a*	*ff*	−0.9	+4.1	...	...	+ 7	− 6
275*b*	*ff*	−0.8	+4.2	...	...	+ 5	+ 3
283	16.26	−1.0	+3.7	...	...	+ 5	−17
299	15.31	−0.9	+0.4	0	+ 2	...	...
306	var.	−0.9	0.0	− 3	+ 1	...	...
310	15.30	−0.9	−1.9	+ 2	+ 2	...	...
311	15.50	−0.9	+0.5	− 9	− 3	...	...
313	15.42	−0.8	+1.6	0	+ 3	...	...

* Rejected in deriving the plate constants.

TABLE III—*Continued*

No.	Pg. Mag.	x	y	80-Foot Focus		25-Foot Focus	
				μ_α	μ_δ	μ_α	μ_δ
Cluster Stars							
323	15.42	−0.′8	−1.′0	−6	+2	....	....
325	15.31	−0.8	−0.4	−2	−4	....	....
326	13.76	−0.8	+3.0	+4	+1	−10	+12
326*a*	*ff*	−0.7	+3.1	....	....	−4	+3
363	15.18	−0.6	−4.0	0	+4	....	....
365	16.04	−0.6	−4.9	....	....	+10	−6
406	15.20	−0.4	−1.4	+2	−2	....	....
418	15.46	−0.4	−2.8	−3	+2	....	....
500	15.44	0.0	+2.2	+4	0	....	....
523	15.31	0.0	+0.7	+1	−3	....	....
559	15.20	+0.2	+0.9	−2	−1	....	....
590	16.20	+0.3	−5.2	....	....	+4	−4
627	16.15	+0.4	+5.0	....	....	−2	−1
641	15.14	+0.5	+3.6	+2	+1	−2	+7
641*a*	*ff*	+0.6	+3.6	....	....	+8	+2
653	15.18	+0.5	+3.2	0	+5	....	....
686	15.42	+0.7	−2.2	0	+1	....	....
690	15.26	+0.7	+0.9	0	0	....	....
712	15.36	+0.8	−1.2	−2	−2	....	....
737	15.20	+0.9	−1.2	+4	−1	....	....
749	16.03	+1.0	+3.5	....	....	+1	+8
775	14.91	+1.0	−2.6	+2	−1	....	....
784	15.31	+1.1	+0.7	+2	−6	....	....
790	16.17	+1.1	−5.6	....	....	+2	+2
791	16.10	+1.1	−4.8	....	....	+6	+1
791*a*	*ff*	+1.2	−5.0	....	....	+15	+4
796	14.93	+1.1	−2.1	+5	0	....	....
804	15.20	+1.2	+0.4	−1	0	....	....
808	15.30	+1.2	+4.2	+8	+4	−11	−3
813	16.22	+1.2	−4.2	....	....	+11	−4
816	var.	+1.2	−0.4	−3	+6	....	....
822*a*	*ff*	+1.2	+5.4	....	....	+4	+5
822	15.09	+1.2	+5.4	+2	+4	+1	+3
829	15.20	+1.3	+0.1	0	+6	....	....
835	13.23	+1.3	−1.9	−4	+2	....	....
837	15.42	+1.3	−1.3	+7	+5	....	....
840	15.50	+1.3	−0.6	+4	−1	....	....
851	15.30	+1.4	−0.3	−1	0	....	....
852	15.17	+1.4	−2.2	+2	−4	....	....
861	14.86	+1.5	−2.4	−1	+1	....	....
873	15.09	+1.6	+1.9	+4	−2	....	....
879	16.20	+1.6	−4.1	....	....	+5	−7
879*a*	*ff*	+1.7	−4.2	....	....	+1	−1
899	15.14	+1.7	+1.8	+2	−2	−4	−2
915	16.58	+1.7	−5.3	....	....	+6	+3
922*a*	*ff*	+1.7	+4.2	....	....	+11	+10
922	16.03	+1.8	+4.2	....	....	+1	+9
925	15.07	+1.8	−2.0	+5	0	....	....
935	14.50	+1.9	+1.2	+4	+3	....	....

TABLE III—*Continued*

No.	Pg. Mag.	x	y	80-Foot Focus μ_α	80-Foot Focus μ_δ	25-Foot Focus μ_α	25-Foot Focus μ_δ
Cluster Stars							
953	15.20	+2′.0	+1′.1	+ 4	+ 2	....	....
978	15.04	+2.2	−0.6	0	− 3	....	....
989	15.60	+2.3	−1.6	− 1	+ 1	....	....
1010	16.26	+2.6	+2.7	....	....	− 1	+ 5
1031	16.07	+2.9	−2.9	....	....	+ 8	+ 5
1034	16.23	+3.0	−5.0	....	....	+ 7	− 6
1034*a*	*ff*	+3.1	−5.1	....	....	+ 4	+ 2
1037*a*	*ff*	+2.9	+3.4	....	....	− 1	+ 5
1037	16.74	+3.0	+3.4	....	....	+11	+ 1
1038*β*	15.07	+3.0	+1.1	+ 3	+ 3	....	....
1042	15.09	+3.0	−0.1	− 2	0	....	....
1063	15.08	+3.7	+1.5	+ 2	+ 4	−18	− 4
1069*a*	*ff*	+3.8	+1.4	....	....	− 1	+ 3
1069	16.26	+4.0	+1.4	....	....	− 1	+ 5
1076	16.27	+4.1	+5.6	....	....	− 1	+ 7
1079	15.14	+4.2	+5.8	+ 3	0	− 2	+ 2
1083*a*	*ff*	+4.2	−2.6	....	....	+12	+ 1
1083	16.10	+4.3	−2.6	....	....	+10	+ 8
1090	16.28	+4.4	+0.8	....	....	+ 3	+ 8
1090*a*	*ff*	+4.6	+0.8	....	....	+11	+ 8
1102	16.06	+5.0	+3.7	....	....	0	− 1
1106*a*	*ff*	+5.2	−4.4	....	....	+22	+14
1106	15.18	+5.4	−4.4	− 1	+ 2	+15	+ 2
1106*b*	*ff*	+5.5	−4.4	....	....	+20	+ 8
1113	15.26	+5.8	+3.5	+ 6	+ 7	−10	− 5

As will be seen later, the internal motions for all three clusters are very small indeed; accordingly, the deviations from the mean motion of a cluster may be used to gain an idea of the accuracy of the measures. This will be an upper limit, since the internal motions, while small, may still amount to a few thousandths of a second of arc, and further, since some stars may have been included which are not members of the clusters. The results for the probable error of an individual μ_α or μ_δ so derived are collected in Table II.

We may conclude from this table that the probable errors in an individual μ_α or μ_δ as derived from plates taken at the 80-foot and 25-foot foci of the 60-inch reflector are, respectively, 0″.030 and 0″.064, divided by the interval in years between the two exposures.

The remaining tables contain the results of the measures. The third and fourth columns in Tables III, V, and VI give the co-ordi-

nates of the stars from the center of the clusters. Table III gives first, in the fifth and sixth columns, the results of the re-reduction of the mean motions derived in *Contribution* No. 284. It is interesting to note that the exclusion of two stars having considerable proper motion has changed the results but slightly, 0.″0015 in each co-ordinate, on the average, for the motion of the cluster stars.

The *ff* stars may be identified by their co-ordinates relative to neighboring stars, which are given in Table IV.

TABLE IV

Positions of the *ff* Stars

No.	Position	No.	Position
49*a*	2.″5 W, 10.″5 S of 49	791*a*	5.″7 E, 11.″1 S of 791
51*a*	6.8 E, 3.8 S of 51	822*a*	4.9 W, 4.1 N of 822
59*a*	6.8 W, 4.9 N of 59	879*a*	7.2 E, 5.2 S of 879
70*a*	6.8 E, 5.3 N of 70	922*a*	4.9 W, 1.7 S of 922
71*a*	3.4 E, 6.4 N of 71	1034*a*	7.1 E, 6.8 S of 1034
82*a*	1.7 W, 4.5 N of 82	1037*a*	3.5 W, 0.4 S of 1037
113*a*	6.6 E, 8.7 N of 113	1069*a*	6.8 W, 3.6 N of 1069
275*a*	11.2 E, 11.9 N of 275	1083*a*	10.0 W, 2.6 S of 1083
275*b*	19.5 E, 15.6 N of 275	1090*a*	7.8 E, 1.6 S of 1090
326*a*	5.4 E, 7.3 N of 326	1106*a*	8.2 W, 0.5 S of 1106
641*a*	6.9 E, 3.1 S of 641	1106*b*	10.0 E, 4.3 S of 1106

Table V gives the results for M 56. The fifth and sixth columns contain the results of the measures of the two pairs of plates in right ascension; the seventh and eighth columns, those in declination; while the last two columns give the means of the measures in the two co-ordinates. The comparison stars are numbered 1 . . . . 65; the cluster stars, 101 . . . . 150.

TABLE V

Co-ordinates and Annual Displacements in M 56

(The Motions Are Given in 0″.001 as a Unit)

No.	Pg. Mag.	x	y	$\mu_{1\alpha}$	$\mu_{2\alpha}$	$\mu_{1\delta}$	$\mu_{2\delta}$	μ_{α}	μ_{δ}
				Comparison Stars					
1……	14.0	−6′.0	+3′.5	+3	−4	+7	+2	0	+4
2……	14.9	−5.9	+0.1	+5	0	0	+6	+2	+3
3……	15.0	−5.7	−1.6	+6	+7	+3	+1	+6	+2
4……	14.1	−5.6	−0.8	+1	−3	+1	−1	−1	0
5……	15.2	−5.2	−3.1	−2	+3	−10	0	0	−5
6……	13.8	−4.4	+1.1	+1	+4	−1	−5	+2	−3
7……	14.5	−4.2	+5.0	−4	−4	−16	−17	−4	−16
8……	15.1	−3.8	−1.6	−1	−2	−2	−4	−2	−3
9……	14.0	−3.7	−0.6	+9	+5	−25	−25	+7	−25
10……	14.4	−3.6	−4.1	+2	+1	+1	+4	+2	+2
11……	15.5	−3.5	−0.6	−2	+3	+1	−4	0	−2
12……	14.9	−3.6	+5.6	+2	+2	+35	+32	+2	+34
13……	13.3	−3.4	+0.7	0	0	−1	−3	0	−2
14……	14.3	−3.4	−4.8	−1	0	+5	+7	0	+6
15……	15.6	−3.4	−0.5	−4	−4	−1	+1	−4	0
16……	15.0	−3.2	−1.5	−7	+3	+8	+8	−2	+8
17……	14.6	−3.1	−3.7	+3	0	−7	−10	+2	−8
18……	13.9	−3.0	+3.1	−10	−11	−16	−16	−10	−16
19……	14.6	−2.8	+1.0	−6	+1	+14	+13	−2	+14
20……	<12	−2.8	+0.2	−3	−3	0	+10	−3	+5
21……	15.0	−2.7	+0.3	−6	−1	+2	+3	−4	+2
22……	14.8	−2.4	−1.9	+5	+5	−2	0	+5	−1
23……	14.7	−2.3	+4.0	−3	−1	−2	−6	−2	−4
24……	15.2	−2.2	+1.9	+1	−4	−1	+3	−2	+1
25……	13.6	−2.0	+3.8	−1	−1	−5	0	−1	−2
26……	13.9	−1.7	−6.5	−2	0	+11	+3	−1	+7
27……	13.5	−1.5	−2.5	−3	+1	+7	+6	−1	+6
28……	15.7	−1.3	−5.0	−1	−8	+6	+4	−4	+5
29……	13.5	−1.1	+6.2	+23	+13	−3	−1	+18	−2
30……	15.3	−1.2	−3.0	0	+6	+6	+3	+3	+4
31……	15.0	−1.0	−5.6	−3	−3	−4	−4	−3	−4
32……	14.9	−0.1	+4.0	+1	−1	−1	−1	0	−1
33……	14.8	+0.1	−3.2	+1	−4	+2	−1	−2	0
34……	15.0	+0.2	+3.2	−7	−4	−16	−17	−6	−16
35……	14.0	+0.5	−2.9	−9	−5	+10	+2	−7	+6
36……	15.2	+0.5	−4.6	+4	−3	+7	+5	0	+6
37……	14.2	+0.8	−4.6	+3	+2	−15	−18	+2	−16
38……	15.5	+1.1	−4.4	+5	−3	+7	+6	+1	+6
39……	12.5	+1.5	+2.6	−7	−1	+4	+8	−4	+6
40……	15.1	+1.5	+4.9	−8	0	−11	−18	−4	−14
41……	14.8	+1.6	+6.0	−2	+2	+5	0	0	+2
42……	14.0	+2.1	+4.8	+2	+4	+1	+5	+3	+3
43……	14.1	+2.4	+4.8	+2	+11	−10	+14	+6	+2
44……	15.9	+2.1	−3.1	0	−2	−9	+6	−1	−2
45……	15.7	+2.3	−5.3	−12	−2	−22	−8	−7	−15
46……	14.9	+2.7	−0.3	−1	0	+12	+10	0	+11
47……	15.4	+2.8	−0.5	−1	−2	+4	+9	−2	+6
48……	13.6	+3.1	−3.6	+2	−1	+15	+4	0	+10

TABLE V—*Continued*

No.	Pg. Mag.	x	y	$\mu_{1\alpha}$	$\mu_{2\alpha}$	$\mu_{1\delta}$	$\mu_{2\delta}$	μ_{α}	μ_{δ}
Comparison Stars									
49.....	15.4	+3′.1	+0′.9	+ 4	− 3	+11	+12	0	+12
50.....	15.6	+3.3	−1.7	+21	+21	− 2	− 4	+21	− 3
51.....	15.9	+3.4	−1.6	+ 7	+ 3	+ 2	− 2	+ 5	0
52.....	15.2	+3.6	+4.6	− 3	− 8	− 6	− 7	− 6	− 6
53.....	15.7	+3.8	+3.4	− 3	0	+ 7	+ 5	− 2	+ 6
54.....	15.8	+3.8	−0.1	+ 9	0	0	−11	+ 4	− 6
55.....	16.0	+4.4	+0.4	− 3	− 3	+ 1	+ 1	− 3	+ 1
56.....	13.7	+4.3	−1.8	− 7	− 5	− 3	− 2	− 6	− 2
57.....	15.5	+4.5	−1.8	+ 6	+ 8	− 8	− 7	+ 7	− 8
58.....	15.3	+4.6	+0.5	+ 5	+ 6	− 6	− 6	+ 6	− 6
59.....	14.6	+4.6	−0.5	+ 2	0	0	+ 1	+ 1	0
60.....	13.8	+4.6	+1.4	− 9	− 3	+ 3	+ 6	− 6	+ 4
61.....	15.0	+5.3	+4.6	0	− 4	+ 4	0	− 2	+ 2
62.....	15.8	+4.8	−5.0	− 2	+ 2	− 2	− 1	0	− 2
63.....	15.4	+5.8	+4.6	− 8	− 6	+ 2	− 5	− 7	− 2
64.....	15.2	+5.4	+3.4	+ 7	− 1	+ 1	+ 3	+ 3	+ 2
65.....	15.5	+5.4	−2.1	− 6	− 4	− 2	+ 3	− 5	0
Cluster Stars									
101.....	13.6	−1′.5	+1′.7	− 1	0	+ 7	+ 8	0	+ 8
102.....	15.1	−1.4	+0.9	− 2	− 5	+ 5	+ 6	− 4	+ 6
103.....	15.6	−1.4	−0.3	− 3	− 2	+12	+ 5	− 2	+ 8
104.....	15.1	−1.3	+0.7	+ 1	− 1	+ 9	+ 5	0	+ 7
105.....	13.0	−1.0	+1.9	+ 5	+ 6	− 5	− 1	+ 6	− 3
106.....	15.7	−1.0	−0.3	− 2	− 5	+ 9	+ 9	− 4	+ 9
107.....	15.9	−1.0	+0.8	− 4	− 9	− 4	+11	− 6	+ 4
108.....	15.9	−0.9	+0.6	0	+ 1	+ 8	+ 8	0	+ 8
109.....	15.6	−0.9	+0.5	0	− 2	+ 9	+12	− 1	+10
110.....	15.0	−0.9	−1.3	0	− 4	+ 3	+ 5	− 2	+ 4
111.....	15.1	−0.7	+0.3	− 5	+ 4	+ 9	+ 9	0	+ 9
112.....	15.8	−0.9	−1.3	− 1	− 3	+ 2	+ 7	− 2	+ 4
113.....	16.3	−0.7	−1.5	+ 5	− 5	+14	+ 6	0	+10
114.....	15.1	−0.6	−0.1	0	+ 5	+ 9	+ 7	+ 2	+ 8
115.....	15.4	−0.6	+0.8	− 8	− 4	+ 9	+12	− 6	+10
116.....	14.4	−0.6	−0.5	+ 6	+ 2	+ 5	+ 8	+ 4	+ 6
117.....	15.2	−0.5	−0.5	− 8	− 3	+11	+12	− 6	+12
118.....	15.9	−0.4	−0.8	− 5	0	+10	+ 9	− 2	+10
119.....	16.2	−0.5	−1.3	−10	− 3	+ 6	+11	− 6	+ 8
120.....	15.2	−0.3	+1.3	− 4	− 3	+ 3	+10	− 4	+ 6
121.....	15.1	−0.3	−0.3	− 2	+ 2	+14	+ 4	0	+ 9
122.....	15.6	−0.3	−0.3	0	+ 2	+ 4	+ 1	+ 1	+ 2
123.....	15.6	−0.3	−0.3	− 8	− 3	+11	+ 9	− 6	+10
124.....	16.2	+0.1	−0.8	+ 2	+ 2	+ 3	+ 4	+ 2	+ 4
125.....	13.1	0.0	+0.5	0	+ 2	+10	+12	+ 1	+11
126.....	15.8	0.0	+0.6	− 8	− 7	− 2	+ 5	− 8	+ 2
127.....	15.4	+0.1	+0.2	− 8	− 4	−12	−10	− 6	−11
128.....	15.0	+0.1	+0.1	+ 2	− 2	+ 3	+10	0	+ 6
129.....	16.0	+0.2	−0.4	− 3	+10	+ 6	+ 4	+ 4	+ 5
130.....	15.8	+0.1	−1.1	0	− 2	+ 4	+ 2	− 1	+ 3

TABLE V—*Continued*

No.	Pg. Mag.	x	y	$\mu_{1\alpha}$	$\mu_{2\alpha}$	$\mu_{1\delta}$	$\mu_{2\delta}$	μ_α	μ_δ
Cluster Stars									
131.....	13.2	+0'.2	−1'.5	+ 4	0	+ 7	+10	+ 2	+ 8
132.....	14.5	+0.1	−2.3	+ 2	0	+15	+ 8	+ 1	+12
133.....	15.5	+0.3	−0.3	+ 3	− 3	+ 7	+14	0	+10
134.....	15.5	+0.3	−0.1	+ 3	+ 4	+ 6	+10	+ 4	+ 8
135.....	16.5	+0.3	+0.7	− 8	− 7	+ 7	− 6	− 8	0
136.....	15.4	+0.3	+0.5	− 2	− 3	+ 4	+11	− 2	+ 8
137.....	14.5	+0.4	+1.5	+ 2	− 2	+ 9	+ 9	0	+ 9
138.....	15.4	+0.5	+2.3	− 5	− 6	+ 4	+10	− 6	+ 7
139.....	15.8	+0.5	+0.1	− 4	− 1	+ 2	+11	− 2	+ 6
140.....	15.3	+0.5	−0.2	− 9	− 2	+12	+ 4	− 6	+ 8
141.....	15.9	+0.7	−0.1	+ 1	− 2	+ 7	+13	0	+10
142.....	15.2	+0.7	+1.7	− 3	− 1	− 3	− 3	− 2	− 3
143.....	16.2	+0.9	+0.3	−10	− 1	+ 6	+15	− 6	+10
144.....	12.7	+0.9	−1.4	− 2	− 1	+11	+ 2	− 2	+ 6
145.....	15.9	+1.0	−1.2	+ 4	− 1	+ 6	+ 4	+ 2	+ 5
146.....	15.9	+1.1	−0.7	− 3	− 3	+ 5	+ 2	− 3	+ 4
147.....	14.8	+1.2	+1.7	− 2	0	+ 9	+18	− 1	+14
148.....	15.5	+1.4	+1.7	− 2	− 1	+ 6	− 2	− 2	+ 2
149.....	13.3	+1.7	−0.5	− 4	− 5	−11	− 7	− 4	− 9
150.....	16.2	+1.9	−0.3	+11	+ 1	+14	+12	+ 6	+13

Table VI shows the results for M 2. The fifth and sixth columns give the results for the 80-foot-focus plates; the last two columns, those for the 25-foot-focus plates. The comparison stars are numbered 1 . . . . 39, but star 17 was excluded from the derivation of the plate constants, as it has a considerable proper motion. The cluster stars are numbered 101 . . . . 224.

TABLE VI

CO-ORDINATES AND ANNUAL DISPLACEMENTS IN M 2

(The motions are given in 0".001 as a unit)

No.	Pg. Mag.	x	y	80-Foot Focus		25-Foot Focus	
				μ_α	μ_δ	μ_α	μ_δ
Comparison Stars							
1	10.6	− 9'.4	− 3'.7	+ 10	+ 6	− 3	+ 6
2	15.2	− 8.4	+ 1.9	− 7	− 2	− 4	+ 3
3	14.9	− 8.3	− 4.6	+ 5	+ 5	+ 6	+ 1
4	15.1	− 7.9	− 4.8	− 8	− 3	− 2	− 1
5	13.3	− 7.7	− 5.4	− 7	− 3	− 12	−12
6	13.4	− 7.1	+ 0.1	+ 4	−21	0	−15
7	15.1	− 6.7	+ 2.9	− 5	−12	+ 2	−12
8	14.4	− 6.2	+ 6.8	− 5	+ 3	− 3	+11
9	16.3	− 4.4	− 2.7	− 13	− 3	+ 8	− 1
10	15.2	− 5.2	+ 5.0	+ 7	+24	+ 3	+14
11	14.8	− 4.5	+ 8.6	− 4	− 1	− 4	− 6
12	13.7	− 3.4	− 8.7	+ 2	− 6	+ 9	+ 7
13	15.3	− 3.3	− 4.5	+ 8	+10	+ 6	+ 3
14	15.0	− 3.4	+ 9.6	+ 9	+19	+ 11	+11
15	15.1	− 1.2	+ 8.4	+ 1	− 6	− 4	− 4
16	15.4	− 0.8	− 6.0	+ 4	+12	− 1	+ 6
17*	13.8	− 0.7	− 7.1	−129	−35	−144	−41
18	13.7	− 0.3	− 8.3	+ 16	0	+ 9	− 2
19	14.3	+ 0.4	− 8.6	+ 2	+ 2	− 12	+ 4
20	14.8	− 1.5	+10.0	+ 3	− 6	+ 5	− 1
21	15.2	+ 1.0	− 5.6	+ 8	+ 8	+ 14	+ 9
22	15.3	+ 1.4	− 9.2	− 13	+ 3	− 7	− 4
23	14.9	+ 1.8	− 4.2	+ 6	+12	+ 6	+12
24	14.8	+ 1.9	+ 8.6	+ 9	− 4	+ 4	− 9
25	15.1	+ 2.3	+ 5.6	− 5	0	− 10	− 2
26	10.3	+ 2.7	+ 3.5	+ 9	− 2	+ 6	+ 6
27	15.2	+ 2.9	− 5.6	+ 11	+ 4	+ 6	+ 8
28	13.6	+ 3.1	+ 6.7	− 11	−17	− 14	−21
29	15.9	+ 4.6	+ 3.3	− 18	−13	− 19	− 8
30	15.6	+ 4.9	− 6.1	− 33	−27	− 26	−23
31	14.7	+ 6.0	− 1.8	+ 5	+ 8	+ 2	+ 1
32	16.1†	+ 6.3	+ 1.5	+ 4	+ 3	+ 24	+ 5
33	14.9	+ 6.2	− 2.4	+ 3	− 1	+ 4	− 2
34	12.7	+ 6.4	− 2.3	− 10	+ 6	− 9	+ 3
35	11.8	+ 7.3	+ 7.7	+ 1	−14	+ 1	− 7
36	13.0	+ 7.7	+ 6.4	+ 1	+17	− 6	+18
37	11.7	+ 7.6	− 7.9	+ 2	− 5	+ 2	+ 2
38	14.0	+10.5	− 4.1	+ 12	+ 2	+ 4	− 6
39	13.5	+10.7	+ 5.2	0	+ 7	+ 7	+ 7
Cluster Stars							
101	15.4	− 3'.0	+ 0'.7	+ 3	+12	+ 2	+ 6
102	15.2	− 3.0	− 0.5	+ 3	+ 4	+ 3	+ 8
103	15.1	− 2.4	+ 1.1	+ 7	+ 5	0	+ 2
104	15.1	− 2.3	+ 1.9	+ 8	+ 8	+ 2	+ 4

TABLE VI—*Continued*

No.	Pg. Mag.	x	y	80-Foot Focus		25-Foot Focus	
				μ_α	μ_δ	μ_α	μ_δ
Cluster Stars							
105......	15.2	− 2.'3	+ 1.'1	+ 8	+ 6	− 7	+ 6
106......	15.4	− 2.2	− 2.6	+ 9	+ 2	+ 7	+ 6
107......	15.3	− 2.0	+ 0.8	+ 8	+ 1	− 1	− 3
108......	15.4	− 1.9	− 1.6	+ 6	+ 9	+ 8	+ 6
109......	15.4	− 1.6	− 1.4	+ 6	− 5	+ 9	+10
110......	15.1	− 1.5	− 1.1	+ 4	+ 2	....	....
111......	15.2	− 1.2	+ 1.0	+ 1	+ 9	....	....
112......	14.8	− 1.0	− 1.0	+ 3	+ 3	....	....
113......	14.2	− 0.8	+ 0.1	+ 4	+ 4	....	....
114......	14.5	− 0.8	+ 1.1	+ 5	+ 9	....	....
115......	14.4	− 0.7	− 0.2	+ 1	+ 6	....	....
116......	14.8†	− 0.7	− 0.3	+ 6	+ 8	....	....
117......	14.2†	− 0.7	+ 0.1	+ 6	+ 6	....	....
118......	14.2	− 0.5	+ 0.5	+ 2	+ 2	....	....
119......	14.2	− 0.4	+ 0.5	+ 7	+ 1	....	....
120......	15.0	− 0.7	− 1.5	+ 4	+ 8	....	....
121......	15.1	− 0.5	− 1.0	+ 2	+10	....	....
122......	15.1	− 0.5	− 1.3	+ 2	+ 4	....	....
123......	14.3†	− 0.5	− 0.5	+ 6	+ 2	....	....
124......	14.3	− 0.4	− 0.3	+ 5	+ 5	....	....
125......	13.7	− 0.3	+ 0.4	+ 8	+ 1	....	....
126......	15.1	− 0.5	− 1.1	+ 8	+ 9	....	....
127......	15.1†	− 0.4	− 1.1	+ 4	− 1	....	....
128......	15.1	− 0.5	− 1.2	+ 2	+ 8	....	....
129......	15.5	− 0.4	− 2.4	− 1	− 1	+ 3	+ 2
130......	15.2	− 0.4	− 2.9	+ 3	+ 1	+ 6	+ 7
131......	15.2	− 0.4	− 3.5	+ 2	+12	+ 9	+ 9
132......	15.1	− 0.3	+ 4.1	+ 5	+ 4	− 1	+ 5
133......	15.0†	− 0.4	+ 2.3	+ 2	+ 6	+ 4	+ 1
134......	14.8	− 0.4	+ 1.8	+ 6	+ 3	− 2	+ 2
135......	14.8	− 0.1	+ 1.4	+ 5	+ 7	....	....
136......	13.8	− 0.1	+ 0.4	+ 8	+ 4	....	....
137......	13.1†	+ 0.1	− 0.5	+ 6	+11	....	....
138......	14.0	0.0	− 0.5	+ 6	+ 8	....	....
139......	14.2†	− 0.1	− 0.7	+ 1	+ 4	....	....
140......	14.7	− 0.1	− 1.1	+ 1	+ 7	....	....
141......	15.1	− 0.1	− 1.5	+ 3	+ 3	....	....
142......	15.1	+ 0.1	− 1.4	+ 6	+ 2	....	....
143......	14.7	+ 0.1	− 0.8	+ 2	+ 7	....	....
144......	13.9	+ 0.1	− 0.6	+ 4	+ 3	....	....
145......	14.7†	+ 0.1	− 0.4	+ 7	+ 3	....	....
146......	14.1†	+ 0.3	− 0.3	+ 5	+ 9	....	....
147......	13.3†	+ 0.4	− 0.3	+ 4	+ 6	....	....
148......	14.4	+ 0.3	+ 0.7	+ 5	+10	....	....
149......	14.5	+ 0.3	+ 0.8	+ 3	+ 5	....	....
150......	14.5	+ 0.3	+ 1.5	+ 3	− 2	....	....
151......	14.6	+ 0.5	+ 1.4	+ 5	+ 5	....	....
152......	15.0	+ 0.7	+ 3.5	+ 6	+ 5	+ 2	+ 1
153......	14.0†	+ 0.5	− 0.1	+ 3	+ 9	....	....

TABLE VI—*Continued*

No.	Pg. Mag.	x	y	80-Foot Focus μ_α	80-Foot Focus μ_δ	25-Foot Focus μ_α	25-Foot Focus μ_δ
			Cluster Stars				
154......	13.6	+0'.6	−0'.3	+5	+6	....	....
155......	13.9†	+0.7	−0.4	+5	+6	....	....
156......	14.9	+0.6	−1.5	+4	+5	....	....
157......	15.1	+0.8	−0.2	−1	+6	....	....
158......	14.5	+0.8	+0.3	+3	+4	....	....
159......	14.8	+0.7	+0.5	+9	+3	....	....
160......	14.0	+1.2	+3.1	+4	+4	0	+1
161......	15.3	+1.0	−2.9	+6	+11	+11	+3
162......	14.8	+1.2	−0.7	+1	+6	....	....
163......	15.0	+1.4	−0.4	+4	+2	....	....
164......	12.7	+1.4	+0.3	+5	+2	....	....
165......	15.4	+1.6	+1.4	+10	+7	+7	+8
166......	14.9	+1.6	−1.0	+6	+8	....	....
167......	15.1	+1.6	−1.4	+9	+4	....	....
168......	15.2	+1.8	−2.0	+2	+6	+3	+7
169......	14.9	+1.9	+1.4	+6	+4	+1	+8
170......	14.7	+2.0	+3.3	+9	+4	−2	+8
171......	15.2	+2.0	+3.7	+7	+2	+3	+7
172......	14.9	+2.2	0.0	+3	+7	+5	+3
173......	15.0	+2.4	−1.8	+3	+9	+6	+10
174......	15.2	+2.9	−1.1	+6	0	+5	+9
175......	13.2	+2.9	−2.9	+17	+6	+9	+1
176......	13.1	+3.7	−2.3	+1	+6	−5	+3
177......	16.2	−1.6	−1.6	....	....	+12	+2
178......	16.7	−1.5	−2.3	....	....	+11	−2
179......	16.0	−1.4	−2.4	....	....	+12	+3
180......	16.2	−1.3	−3.3	....	....	+8	+3
181......	16.2	−1.0	−2.9	....	....	+12	0
182......	16.0	−1.2	−1.7	....	....	+9	+7
183......	16.0	−1.5	+2.4	....	....	+12	+1
184......	16.1	−1.3	+2.2	....	....	+11	+3
185......	16.0	−1.3	+2.4	....	....	+12	+3
186......	15.6	−0.8	+3.2	....	....	+1	+4
187......	17.0	−0.7	+3.3	....	....	+14	+3
188......	15.8	−0.6	+3.5	....	....	+10	+5
189......	>17	−0.8	+4.7	....	....	+19	+3
190......	>17	−0.7	+4.6	....	....	+16	+11
191......	>17	−1.2	−4.2	....	....	+14	−1
192......	16.8	−1.0	−4.5	....	....	+20	−7
193......	16.3	−1.0	−4.7	....	....	+14	−1
194......	>17	−0.8	−4.7	....	....	+17	−7
195......	17.0	−0.3	−3.6	....	....	+19	−3
196......	16.3	0.0	+2.3	....	....	+9	−3
197......	16.2	0.0	+2.5	....	....	+6	+8
198......	16.0	0.0	+2.7	....	....	+7	+3
199......	16.7	+1.5	−4.8	....	....	+23	−6
200......	16.8	+2.0	−3.7	....	....	+18	0
201......	16.2	+2.2	−3.6	....	....	+14	−2
202......	16.7	+2.3	−4.9	....	....	+10	−22

TABLE VI—*Continued*

No.	Pg. Mag.	x	y	80-Foot Focus		25-Foot Focus	
				μ_α	μ_δ	μ_α	μ_δ
			Cluster Stars				
203......	16.6	+ 2′.3	− 5′.2	....	....	+ 19	+ 5
204......	17.0	+ 2.7	− 4.8	....	....	+ 34	− 8
205......	17.0	+ 2.2	− 3.0	....	....	+ 27	− 3
206......	16.6	+ 2.5	− 3.0	....	....	+ 18	+ 1
207......	16.6	+ 1.9	+ 4.1	....	....	+ 1	+ 7
208......	>17	+ 2.1	+ 3.8	....	....	+ 21	− 2
209......	>17	+ 0.2	+ 4.2	....	....	+ 4	0
210......	16.7	+ 0.3	+ 4.1	....	....	+ 2	+ 3
211......	16.6	+ 0.6	+ 4.3	....	....	+ 11	+11
212......	16.4	+ 0.7	+ 5.5	....	....	+ 14	+ 7
213......	>17	+ 0.8	+ 5.4	....	....	+ 2	− 1
214......	15.7	+ 1.3	+ 4.9	....	....	+ 1	+ 4
215......	>17	+ 1.1	+ 6.4	....	....	− 6	− 3
216......	16.8	+ 1.6	+ 6.4	....	....	+ 8	+ 7
217......	16.9	+ 1.4	+ 5.8	....	....	+ 3	+ 6
218......	16.8	− 2.4	+ 4.1	....	....	+ 9	+ 2
219......	16.9	− 2.5	+ 3.8	....	....	+ 12	+ 3
220......	17.0	− 3.4	+ 1.7	....	....	+ 9	− 4
221......	16.6	− ..5	+ 1.0	....	....	+ 13	+ 3
222......	16.9†	− 2.6	+ 3.5	....	....	+ 8	− 2
223......	>17	+ 4.7	+ 0.7	....	....	+ 16	+ 9
224......	17.0	+ 5.1	+ 0.8	....	....	+ 26	+ 7

* Rejected in determining the plate constants.
† Magnitude uncertain.

The next step is the derivation of the total motions of the clusters. Because of the nearly symmetrical distribution of the stars around the center, we may take the straight mean of the motions. In this way we find the results given in Table VII. The agreement between

TABLE VII

RELATIVE PROPER MOTIONS OF CLUSTERS

Object	Plates, with Focus in Feet	μ_α	μ_δ
M 13..........	Two pairs 80	+0″.0008	+0″.0004
	One pair 25	+0.0028	+0.0008
M 56..........	Two pairs 80	−0.0018	+0.0066
M 2...........	One pair 80	+0.0046	+0.0050
	One pair 25	+0.0091	+0.0027

the values derived from the 80-foot- and 25-foot-focus plates is satisfactory. Taking into account the weights of the different sets of plates, we adopt the mean relative motions:

M 13	$\mu_\alpha = +0''.0015$	$\mu_\delta = +0''.0005$
M 56	$\mu_\alpha = -0.0018$	$\mu_\delta = +0.0066$
M 2	$\mu_\alpha = +0.0061$	$\mu_\delta = +0.0042$

These motions are referred to the mean of the comparison stars. With the help of Table 26 of *Groningen Publications*, No. 29, we can compute the mean parallactic motions of the comparison stars used. For the three clusters, M 13, 56, and 2, they are $0''.0010$ in $\chi = 286°$, $0''.0005$ in $\chi = 94°$, and $0''.0027$ in $\chi = 127°$, respectively. The final absolute motions of the clusters may therefore be taken as:

M 13	$\mu_\alpha = +0''.0005$	$\mu_\delta = +0''.0008$
M 56	-0.0013	$+0.0066$
M 2	$+0.0082$	$+0.0026$

Although the number of clusters is but three, it seems worth while to derive from these motions an indication of their mean parallax by the use of the formula:

$$\bar{\pi} = \frac{4.74}{V} \cdot \frac{\bar{\mu}}{\sqrt{2}} .$$

Using Strömberg's recent value, $V = 329$ km/sec.,[1] we find $\bar{\pi} = 0''.000061$, which compares favorably with the mean of the three parallaxes found by Shapley, viz., $0''.000065$.

Subtracting the values of the mean motions given in Table VII from the proper motions in Tables III, V, and VI, we derive the internal motions of the clusters given in the second and third columns of Tables VIII, IX, and X. The unit for the motions is again $0''.001$.

[1] *Mt. Wilson Contr.*, No. 292; *Astrophysical Journal*, **61**, 353, 1925.

TABLE VIII

Internal Motion in M 13

No.	μ_α	μ_δ	$\mu_{rad.}$	$\mu_{tang.}$	No.	μ_α	μ_δ	$\mu_{rad.}$	$\mu_{tang.}$
80-Foot-Focus Plates									
2	+10	−3	−10	+2	686	−1	+1	−1	+1
7	+4	−2	−4	+2	690	−1	0	0	−1
17	+1	−1	0	−1	712	−3	−2	0	+4
25	+2	+1	−1	+2	737	+3	−1	+3	−2
46	−10	+1	+9	+4	775	+1	−1	+1	0
49	+3	−3	−3	−3	784	+1	−6	−4	+5
59	+6	−5	−8	+1	796	+4	0	+2	−4
70	−6	+1	+5	−3	804	−2	0	−2	−1
72	−7	−1	+7	+3	808	+7	+4	+5	+6
96	−4	−1	+4	−1	816	−4	+6	−4	−6
118	+3	−3	−3	−3	822	+1	+4	+4	+1
158	−3	−3	0	−4	829	−1	+6	+1	−6
222	−5	+1	+3	+4	835	−5	+2	−4	+3
246	−4	+4	+1	+5	837	+6	+5	+1	−8
252	+1	+1	0	+1	840	+3	−1	+3	0
299	−1	+2	+2	+1	851	−2	0	−2	0
306	−4	+1	+4	0	852	+1	−4	+4	+1
310	+1	+2	−2	+1	861	−2	+1	−2	+1
311	−10	−3	+6	−8	873	+3	−2	0	+4
313	−1	+3	+3	+1	899	+1	−2	−1	+2
323	−7	+2	+5	+6	925	+4	0	+3	−3
325	−3	−4	+3	−4	935	+3	+3	+4	0
326	+3	+1	0	+3	953	+3	+2	+3	+1
363	−1	+4	−4	+2	978	−1	−3	−1	+3
406	+1	−2	+1	−2	989	−2	+1	−2	0
418	−4	+2	−1	+4	1038β	+2	+3	+3	−2
500	+3	0	0	+3	1042	−3	0	−3	0
523	0	−3	−3	−1	1063	+1	+4	+3	−3
559	−3	−1	−1	−3	1079	+2	0	+1	+2
641	+1	+1	+1	+1	1106	−2	+2	−3	0
653	−1	+5	+5	−1	1113	+5	+7	+8	−3
25-Foot-Focus Plates									
2	−8	+2	+8	−1	71	−4	+1	+4	0
7	+7	+3	−3	+7	71a	+2	−3	−3	−2
17	−7	−4	+8	+1	72	−7	−5	+9	0
21	−3	+4	+5	+1	76	−10	0	+9	+4
25	+1	0	−1	0	82a	+6	−8	−10	−1
46	−12	−5	+13	0	82	−5	−14	−5	−14
49a	+11	−3	−11	−3	96	−13	+1	+13	−1
49	−4	+1	+4	+1	113	+8	−5	0	−9
51	−1	+1	+1	+1	113a	+11	−2	−4	−10
51a	+5	0	−5	+1	155	+4	+1	−2	−3
54	−4	−6	−1	−7	158	+1	−11	−8	−7
59a	+3	0	−2	+2	159	−3	−8	+8	−1
59	−4	0	+3	−3	236	−11	+4	+7	−9
70	−4	+1	+4	−2	275	−2	0	0	−2
70a	+3	−8	−7	−4	275a	+4	−7	−7	+3

TABLE VIII—*Continued*

No.	μ_α	μ_δ	$\mu_{rad.}$	$\mu_{tang.}$	No.	μ_α	μ_δ	$\mu_{rad.}$	$\mu_{tang.}$
				25-Foot-Focus Plates					
275*b*	+2	+2	+2	+2	922	−2	+8	+6	−5
283	+2	−18	−18	−3	1010	−4	+4	+1	−6
326	−13	+11	+14	−10	1031	+5	+4	+1	−6
326*a*	−7	+2	+3	−6	1034	+4	−7	+8	0
365	+7	−7	+6	−8	1034*a*	+1	+1	0	−1
590	+1	−5	+5	−1	1037*a*	−4	+4	+1	−6
627	−5	−2	−2	−5	1037	+8	0	+5	+6
641	−5	+6	+6	−5	1063	−21	−5	−21	−5
641*a*	+5	+1	+2	+5	1069*a*	−4	+2	−3	−3
749	−2	+7	+6	−4	1069	−4	+4	−2	−5
790	−1	+1	−1	+1	1076	−4	+6	+2	−7
791	+3	0	+1	−3	1079	−5	+1	−2	−5
791*a*	+12	+3	0	−12	1083	+9	0	+8	−4
808	−14	−4	−7	−13	1083*a*	+7	+7	+2	−10
813	+8	−5	+6	−7	1090	0	+7	+2	−7
822*a*	+1	+4	+4	0	1090*a*	+8	+7	+9	−5
822	−2	+2	+2	−2	1102	−3	−2	−3	−1
879	+2	−8	+8	+1	1106*a*	+19	+13	+6	−22
879*a*	−2	−2	+2	+2	1106	+12	+1	+9	−8
899	−7	−3	−6	−4	1106*b*	+17	+7	+9	−16
915	+3	+2	−1	−3	1113	−13	−6	−14	−2
922*a*	+8	+9	+11	+4					

The resulting values have been used for different purposes: (*a*) They were utilized to derive an upper limit of the probable errors; the results have been given on page 92. (*b*) In *Contribution* No. 129,[1] Pease and Shapley have called attention to the asymmetry found in several globular clusters. While this asymmetry is extremely small, if not negligible, for M 56, they were able to derive "galactic planes" for both M 13 and 2. From the flattening of several of the clusters, as well as for theoretical reasons, we might expect the stars in the clusters to be moving in orbits parallel to the galactic planes, around the center of gravity; but according to Russell such motions should be less than 0".001 per year even in the nearest of clusters. The internal motions for the two clusters, M 13 and 2, were analyzed into components, parallel to and at right angles to these planes. In both cases the positive sign was used for motion in

[1] *Astrophysical Journal*, **45**, 225, 1917.

the direction southeast and away from the "galactic plane," respectively. The results are:

M 13

Two pairs of plates, 80-foot focus $\begin{cases} \mu \parallel = -0\overset{''}{.}0004 \pm 0\overset{''}{.}0003 \\ \mu \perp = +0.0005 \pm 0.0003 \end{cases}$

One pair of plates, 25-foot focus $\begin{cases} \mu \parallel = -0.0001 \pm 0.0005 \\ \mu \perp = -0.0008 \pm 0.0005 \end{cases}$

M 2

One pair of plates, 80-foot focus $\begin{cases} \mu \parallel = -0.0003 \pm 0.0002 \\ \mu \perp = +0.0002 \pm 0.0002 \end{cases}$

One pair of plates, 25-foot focus $\begin{cases} \mu \parallel = +0.0004 \pm 0.0006 \\ \mu \perp = -0.0004 \pm 0.0005 \end{cases}$

The dispersion of the motions in the direction parallel to and at right angles to the "galactic plane" is practically the same; this indicates

TABLE IX

INTERNAL MOTION IN M 56

No.	μ_α	μ_δ	$\mu_{rad.}$	$\mu_{tang.}$	No.	μ_α	μ_δ	$\mu_{rad.}$	$\mu_{tang.}$
101	+2	+1	0	+2	126	−6	−5	−7	−4
102	−2	−1	+1	−2	127	−4	−18	−17	+7
103	0	+1	0	+1	128	+2	−1	+1	+2
104	+2	0	−1	+2	129	+6	−2	+6	+2
105	+8	−10	−12	+4	130	+1	−4	+4	0
106	−2	+2	+2	+2	131	+4	+1	−1	−4
107	−4	−3	0	−5	132	+3	+5	−4	−4
108	+2	+1	−1	+2	133	+2	+3	+2	−3
109	+1	+3	+1	+3	134	+6	+1	+6	+1
110	0	−3	+2	−2	135	−6	−7	−9	−2
111	+2	+2	0	+3	136	0	+1	+1	0
112	0	−3	+2	−2	137	+2	+2	+3	+1
113	+2	+3	−4	−1	138	−4	0	−1	−4
114	+4	+1	−3	+3	139	0	−1	−1	+1
115	−4	+3	+4	−3	140	−4	+1	−4	−1
116	+6	−1	−3	−5	141	+2	+3	+3	−2
117	−4	+5	−1	+6	142	0	−10	−9	+4
118	0	+3	−3	+1	143	−4	+3	−1	−5
119	−4	+1	0	+4	144	0	−1	+1	+1
120	−2	−1	−1	−2	145	+4	−2	+4	−1
121	+2	+2	−1	+3	146	−1	−3	0	+3
122	+3	−5	−2	−5	147	+1	+7	+6	−3
123	−4	+3	+5	+1	148	0	−5	−4	+3
124	+4	−3	+5	−2	149	−2	−16	+1	+16
125	+3	+4	+5	+2	150	+8	+6	+8	−6

that there is no pronounced motion parallel to the galactic planes. (*c*) The internal motions for all three clusters were finally resolved

TABLE X

Internal Motion in M 2

No.	μ_α	μ_δ	$\mu_{rad.}$	$\mu_{tang.}$	No.	μ_α	μ_δ	$\mu_{rad.}$	$\mu_{tang.}$
				80-Foot-Focus Plates					
101	−2	+7	+3	+6	139	−4	−1	+1	+4
102	−2	−1	+2	−1	140	−4	+2	−1	+4
103	+2	0	−1	+1	141	−2	−2	+2	+2
104	+3	+3	0	+4	142	+1	−3	+3	−1
105	+3	+1	−3	+2	143	−3	+2	−2	+3
106	+4	−3	0	−5	144	−1	−2	+2	+1
107	+3	−4	−4	−3	145	+2	−2	+3	0
108	+1	+4	−4	+2	146	0	+4	−2	−3
109	+1	−10	+6	−8	147	−1	+1	−1	−1
110	−1	−3	+3	−1	148	0	+5	+4	−2
111	−4	+4	+6	+1	149	−2	0	−1	−2
112	−2	−2	+3	0	150	−2	−7	−7	−1
113	−1	−1	+1	−1	151	0	0	0	0
114	0	+4	+3	+2	152	+1	0	0	−1
115	−4	+1	+3	+2	153	−2	+4	−3	−3
116	+1	+3	−2	+3	154	0	+1	−1	−1
117	+1	+1	−1	+1	155	0	+1	−1	−1
118	−3	−3	−1	−4	156	−1	0	0	−1
119	+2	−4	−4	−2	157	−6	+1	−6	0
120	−1	+3	−2	+2	158	−2	−1	−2	−1
121	−3	+5	−3	+5	159	+4	−2	+3	+3
122	−3	−1	+2	+3	160	−1	−1	−1	0
123	+1	−3	+1	−3	161	+1	+6	−5	−4
124	0	0	0	0	162	−4	+1	−4	+1
125	+3	−4	−5	+1	163	−1	−3	0	+3
126	+3	+4	−5	−2	164	0	−3	−1	+3
127	−1	−6	+6	−1	165	+5	+2	+5	+2
128	−3	+3	−3	+3	166	+1	+3	0	−3
129	−6	−6	+7	+5	167	+4	−1	+4	−1
130	−2	−4	+4	+1	168	−3	+1	−2	+2
131	−3	+7	−7	+3	169	+1	−1	0	+1
132	0	−1	−1	0	170	+4	−1	+2	+4
133	−3	+1	+2	−2	171	+2	−3	−4	−1
134	+1	−2	−2	+1	172	−2	+2	−2	−1
135	0	+2	+2	0	173	−2	+4	−4	−2
136	+3	−1	−1	+3	174	+1	−5	+2	+5
137	+1	+6	−6	−2	175	+12	+1	+8	−9
138	+1	+3	−3	−1	176	−4	+1	−4	+1
				25-Foot-Focus Plates					
101	−7	+3	+7	+1	109	0	+7	−4	+5
102	−6	+5	+5	+6	129	−6	+9	−7	+8
103	−9	−1	+8	−4	130	−3	+4	−3	+4
104	−7	+1	+6	−3	131	0	+6	−6	+1
105	−16	+3	+16	−4	132	−10	+2	+3	−10
106	−2	+3	−1	+3	133	−5	−2	−1	−5
107	−10	−6	+7	−9	134	−11	−1	+2	−11
108	−1	+3	−1	+3	152	−7	−2	−3	−7

TABLE X—*Continued*

No.	μ_α	μ_δ	$\mu_{rad.}$	$\mu_{tang.}$	No.	μ_α	μ_δ	$\mu_{rad.}$	$\mu_{tang.}$
25-Foot-Focus Plates									
160	−9	−2	−5	−8	195	+10	−6	+5	−11
161	+2	0	+1	−2	196	0	−6	−6	0
165	−2	+5	+2	−5	197	−3	+5	+5	−3
168	−6	+4	−7	+2	198	−2	0	0	−2
169	−8	+5	−4	−9	199	+14	−9	+13	−11
170	−11	+5	−2	−12	200	+9	−3	+7	−6
171	−6	+4	0	−7	201	+5	−5	+7	−2
172	−4	0	−4	0	202	+1	−25	+23	+9
173	−3	+7	−6	−4	203	+10	+2	+3	−10
174	−4	+6	−6	−4	204	+25	−11	+21	−17
175	0	−2	+2	+1	205	+18	−6	+15	−12
176	−14	0	−12	+7	206	+9	−2	+7	−6
177	+3	−1	−2	−3	207	−8	+4	0	−9
178	+2	−5	+3	−5	208	+12	−5	+2	+13
179	+3	0	−1	−3	209	−5	−3	−3	−5
180	−1	0	+1	0	210	−7	0	−7	+1
181	+3	−3	+2	−4	211	+2	+8	+8	+1
182	0	+4	−3	+2	212	+5	+4	+4	+5
183	+3	−2	−3	+2	213	+3	−4	−3	+4
184	+2	0	−1	+2	214	−8	+1	−1	−8
185	+3	0	−1	+3	215	−15	−6	−8	−14
186	−8	+1	+3	−8	216	−1	+4	+4	−2
187	+5	0	−1	+5	217	−6	+3	+2	−6
188	+1	+2	+2	+1	218	0	−1	−1	−1
189	+10	0	−2	+10	219	+3	0	−2	+2
190	+7	+8	+6	+9	220	0	−7	−3	−6
191	+5	−4	+2	−6	221	+4	0	−3	+2
192	+11	−10	+7	−13	222	−1	−5	−3	−4
193	+5	−4	+3	−6	223	+7	+6	+8	−5
194	+8	−10	+8	−10	224	+17	+4	+17	−2

into radial and tangential components, the positive signs being used for motion outward and in the direction N-E-S-W. The results are given in the fourth and fifth columns of Tables VIII, IX, and X, using again $0\overset{''}{.}001$ as a unit.

A radial motion might be expected if the clusters were either expanding or contracting. For M 13 we find from the 80-foot-focus plates the mean radial motion $+0\overset{''}{.}0006 \pm 0\overset{''}{.}0003$, and from the 25-foot-focus plates $+0\overset{''}{.}0013 \pm 0\overset{''}{.}0005$; for M 56 we find $-0\overset{''}{.}0002 \pm 0\overset{''}{.}0005$; for M 2 we find $-0\overset{''}{.}0004 \pm 0\overset{''}{.}0003$ from the 80-foot-focus plates and $+0\overset{''}{.}0019 \pm 0\overset{''}{.}0005$ from the 25-foot-focus plates. While there is slight preponderance of the positive sign, indicating an expansion of the clusters, the amount is too small to draw any definite conclusions.

From (*b*) it follows that the motions resulting from a possible rotation of the clusters are small. Tangential components of the motions were derived therefore only because evidence of such motions had been found in the measures of spiral nebulae. The results for the clusters are:

M 13.....	Two pairs of 80-foot-focus plates	$\mu_{\text{tang.}} = +0''.0001 \pm 0''.0003$
	One pair of 25-foot-focus plates	$\mu_{\text{tang.}} = -0.0034 \pm 0.0004$
M 56.....	Two pairs of 80-foot-focus plates	$\mu_{\text{tang.}} = -0.0002 \pm 0.0003$
M 2......	One pair of 80-foot-focus plates	$\mu_{\text{tang.}} = +0.0001 \pm 0.0002$
	One pair of 25-foot-focus plates	$\mu_{\text{tang.}} = -0.0026 \pm 0.0005$

In the mean, the tangential component for all the cluster plates taken at the 80-foot focus is exactly 0''.0000, while in the case of M 33, observed at that focus, an annual tangential component of 0''.0140 was found; for the cluster plates taken at the 25-foot focus the mean tangential component for M 13 and 2 is 0''.0030, while for seven spirals it is 0''.0184.

It has been thought that the large displacements in the spirals might be due to a difference in quality in the old and the new plates. In that case the annual motions derived would be smaller for pairs of plates with longer intervals, but the total displacements would be of the same order, and should be the same for both the clusters and the spirals. We find, however, that the total tangential displacement in the case of the 80-foot-focus plates is 0''.070 for the spirals, 0''.000 for the clusters, and, in the case of the 25-foot-focus plates, 0''.174 for the spirals and 0''.044 for the clusters.

The length of exposure, too, cannot have caused the displacements in the spirals; for all the 80-foot-focus plates, for M 33 as well as for the clusters, these were about a half-hour, except for M 2, where the exposure times were fifteen and twenty minutes. The exposures for the 25-foot-focus plates of the spirals ranged from one hour to eight hours, while for the clusters they were one hundred minutes and five hours, respectively.

Since the comparison stars for the spirals are brighter, on the whole, than the nebular points measured, it has been suggested that the rotational displacements found in the spirals might be due to some obscure magnitude error. It was for this reason that several

faint objects were measured on the plates of the clusters taken at the 25-foot focus. While no stars fainter than magnitude 16.5 could be measured on the plates taken at the 80-foot focus, the range in magnitude for the 25-foot focus plates was from 12.5 to the eighteenth or nineteenth magnitude for M 13, and from 10.3 to well below the seventeenth magnitude for M 2. No correlation whatsoever

TABLE XI

Object	Magnitude							
	<12	13	14	15	16	17	18–19	
				80-Foot-Focus Plates				
M 13*	....	0 (9)	+1 (13)	−1 (14)	−1 (12)	....	....	Comparison stars
		+1 (8)	−1 (7)	0 (45)	....	....	....	Cluster stars
M 56	....	−1 (12)	0 (21)	0 (32)	....	....	....	Comparison stars
		$+3^5$(6)†	−3 (4)	$+0^5$(33)	-1^5(7)	....	....	Cluster stars
M 2‡	+4(5)	−9 (7)	−2 (10)	+3 (14)	0 (2)	....	....	Comparison stars
	....	0 (7)	0 (32)	$+0^5$(34)	....	....	....	Cluster stars
				25-Foot Focus Plates				
M 13*	....	-0^5(8)	$+0^5$(13)	0 (14)	−1 (11)	....	....	Comparison stars
		−4 (5)	0 (3)	−3 (12)	−3 (30)	....	-3^5(22)	Cluster stars
M 2‡	+2(5)	−8 (7)	+1 (10)	+1 (14)	-2^5(2)	....	....	Comparison stars
	....	....	−8 (5)	−2 (24)	−2 (30)	−3 (15)	....	Cluster stars

* The two variable stars, Nos. 306 and 816, and the two stars having considerable proper motion, *p* and *s*, were excluded.

† It is doubtful if these six stars, Nos. 101, 105, 125, 131, 144, and 149, are members of the cluster.

‡ Star No. 17 was excluded on account of its large proper motion; Nos. 164, 175, and 176, on account of the uncertainty as to their membership in the cluster.

seems to exist between magnitude and tangential component, as may be seen from Table XI, where the tangential components are given in thousandths of a second of arc, while the figures in parentheses indicate the number of stars used.

In concluding I wish to express my thanks to Messrs. Pease and Humason for obtaining the 25-foot-focus plates, and to Mrs. Marsh, of the computing division, who has assisted in the numerous reductions involved.

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ON THE PERIOD OF 27 CANIS MAJORIS

By OTTO STRUVE

ABSTRACT

Since this southern star can be observed at the Yerkes Observatory only when near the meridian, there are *theoretically three periods* which will satisfy the observations: one is the long period of 1165 days, while the other two periods are equal to one sidereal day *plus* or *minus* 1 minute and 14 seconds. Check observations taken during the same night permit one to discard the longer one of the two short periods. The shorter one remains possible, although there is some evidence against it. The appearance of the absorption lines is not what would be expected in a binary of such short period. There are no other truly double stars of spectral type B with periods shorter than 1.3 days. The *eccentricity of* 27 Canis Majoris would seem to be greater than would be expected in so close a binary. On the other hand, the large mass resulting from the long period is so improbable that the *possibility of a one-day period* must be kept under consideration.

The question could be solved by means of observations taken in different hour-angles during the same night and covering a sufficiently long interval of time. Such observations could best be made at a southern observatory, where the star rises high enough above the horizon.

In my paper[1] in the June issue of this *Journal* on the spectroscopic binary 27 Canis Majoris I came to the conclusion[2] that the observations could be reconciled with either one of three seemingly impossible alternatives: (1) The mass is incredibly large. (2) There exist unknown effects which distort the measured radial velocities. (3) The period is almost precisely one day.

Professor B. P. Gerasimovič has expressed to me the opinion that the third alternative appears to be the least improbable, since it is much easier to imagine the existence of a star having a period $0\overset{d}{.}3$ shorter than that of any other real B-type double star than to grant the existence of a system with a mass nearly one thousand times larger than that of the sun. With this idea in mind, I have once more gone over the whole material in order to compare the various possibilities.

POSSIBILITY OF SHORT PERIOD

As I have stated in my first paper,[3] the difficulty in determining the correct period arises from the fact that the star, having a declination of $-26°$, can be observed in northern latitudes only when near

[1] *Astrophysical Journal*, **65**, 273, 1927.

[2] *Ibid.*, p. 285. [3] *Ibid.*, p. 277.

the meridian. Under such conditions there always exist three distinct periods that will fit all observations equally well. In connection with variable stars this was pointed out almost simultaneously by C. Hoffmeister[1] and Harlow Shapley.[2] The relationship of the three periods will hold, of course, not only in case of one long period and two short ones, but also in cases where all three periods are roughly of the same order of magnitude. J. G. Hagen[3] has shown that the three periods $P > p_1 > 1^d > p_2$ and their respective phases Φ, ϕ_1 and ϕ_2 (expressed in units of the period) are related to one another in the following way:

$$\begin{array}{lll} \Phi = -\phi_1 & p_1 = P/(P-1) & P = p_1/(p_1-1) \\ \Phi = +\phi_2 & p_2 = P/(P+1) & P = p_2/(1-p_2) \\ \phi_1 = -\phi_2 & p_1 = p_2/(2p_2-1) & p_2 = p_1/(2p_1-1) \end{array}$$

For spectroscopic binaries these relations are also true. But here it is customary to observe in equal hour-angles, and not, as is frequently the case for variable star observers, at the same hour of mean time. Therefore, in spectroscopic binaries the three relations will hold if P, p_1, and p_2 are expressed in sidereal time. To illustrate this point, I have computed the phases of all observations of 27 Canis Majoris, for the two periods p_1 and p_2. A simple computation shows that $p_1 = 1.0008591$ sidereal days $= 0.9981291$ mean solar days and $p_2 = 0.9991423$ sidereal days $= 0.9964123$ mean solar days.[4] Figure 1 shows both velocity curves. They have approximately the same shape as the curve derived with the long period of 1165 days[5] and are both algebraically possible. This is, as we have seen, a mathematical necessity due to the observations having been made near the meridian.

CHECK OBSERVATIONS

The question arises next, whether, from the material at hand, it is possible to decide in favor of one of the three curves. In order to check the possibility of a short period, I have taken on each of two

[1] *Astronomische Nachrichten*, **196**, 399, 1913.

[2] *Ibid.*, p. 417, 1913.

[3] "*Die veränderlichen Sterne*" **1**, 624, 1920.

[4] The two short periods p_1 and p_2 are equal to one sidereal day *plus* or *minus* about 1 minute and 14 seconds.

[5] *Astrophysical Journal*, **65**, 278, Fig. 1, 1927.

nights more than one plate. On February 23, 1927, three plates were obtained covering an interval of 0.ᵈ10, while on March 4, 1927, the interval was 0.ᵈ05. To these should be added two more pairs of plates, which, although not taken on the same night, were obtained in very different hour-angles. The first pair is that of March 5 and 8, 1927, covering an interval of 0.ᵈ12, and the second is the pair of Lick observations of December 5 and 24, 1908, covering also an interval of 0.ᵈ12 in phase. The data for these four sets are given in Table I; by means of their phases they can easily be located in Figure 1.

TABLE I

CHECK OBSERVATIONS

Date	Velocity	Phase p_1	Phase p_2
1908 Dec. 5.47.........	+96.2	0.77	0.00
1908 Dec. 24.29.........	+89.2	.63	.88
1927 Feb. 23.104........	−63.3	.875	.586
1927 Feb. 23.151........	59.7	.922	.633
1927 Feb. 23.199........	56.0	.970	.681
1927 Mar. 4.069........	81.3	.857	.583
1927 Mar. 4.119........	74.8	.907	.633
1927 Mar. 5.064........	81.3	.854	.582
1927 Mar. 8.176........	−78.0	0.971	0.704

In all four cases the differences in velocity are negligible, and are fully accounted for by errors of measurement. The direction of change for the last pair is opposite to that of the preceding two sets. The evidence from these plates was at first regarded as sufficient for discarding the short periods. This belief was strengthened by the fact that two other spectroscopic binaries which we observed here last year, π Scorpii[1] and 36 τ^1 Eridani,[2] both of which have periods of the order of one day, easily showed the change in velocity on plates taken during the same night. Both stars are placed equally unfavorably in the sky, having declinations of −26° and −24°, respectively. The lines of π Scorpii are much more difficult to measure than those of 27 Canis Majoris.

[1] Orbit by Struve and Elvey; unpublished.

[2] *Astrophysical Journal*, **65**, 300, 1927.

If the four sets of check observations are identified on the velocity curves of Figure 1, it is apparent that their slopes do not fit the curves satisfactorily. This is especially evident for the lower curve ($p_1 = 0^d.9981291$), for which the general scattering is also very large. I believe, therefore, that we are entitled to discard this period altogether.

For the upper curve ($p_2 = 0^d.9964123$) this is not so evident. The three observations of February 23, 1927, show no slope at all, while

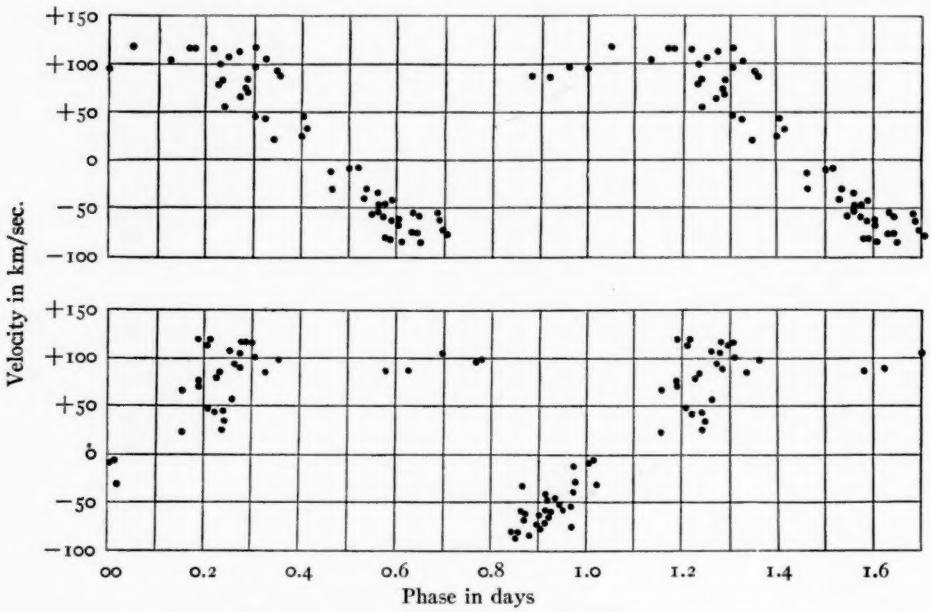

FIG. 1.—Velocity curve of 27 Canis Majoris. The upper curve is reduced with $p_2 = 0^d 9964123$ and the lower with $p_1 = 0^d 9981291$.

the curve seems to demand a gradual decrease in velocity. But if we are willing to reconcile ourselves to the large general scattering of the individual observations, these check plates cannot be regarded as fatal to the short period p_2.

The plates of 1925–1926 were all measured twice. Except for plates designated as "poor" the double measures agree very well with one another, and it is difficult to explain the large residuals from the curve. In the case of the long period this difficulty was

avoided by introducing a shorter oscillation of 120 days—a case that has a counterpart in the spectroscopic binary 7 ϵ Aurigae.[1] But I do not regard this as a sufficient reason for excluding the period p_2.

If I disregard the short oscillation of 120 days, the representation of the observations is about the same whether I use P or p_2. I have computed the values of $\Sigma\Delta^2$, Δ being taken from a smooth curve drawn through the observations. Within the errors of estimation, the values are the same. It is thus impossible, from this point of view, to decide on the period. The third value, p_1, gives a much larger value for $\Sigma\Delta^2$.

LINE-WIDTHS

It is generally believed that very close double stars show wide and hazy lines due to the rapid rotation of the components.[2] This is the case in all spectroscopic binaries with sufficiently large inclinations. The most typical example is W Ursae Majoris, but the same effect is also present in all other stars where it should theoretically exist. An interesting example is π Scorpii, where $P = 1^{\mathrm{d}}.571$ and $K_1 = 138$ km/sec. Entirely different lines are found in 27 Canis Majoris. Those of hydrogen are sharp; those of helium, while wide, are not more so than is usually the case in B-type stars (width about 1.8 A), while the stellar calcium lines are distinctly narrow (width about 0.8 A). The appearance of the absorption lines would thus be construed as contradictory to the existence of a short period.

FREQUENCY-DISTRIBUTION OF P AMONG DOUBLE STARS

I have stated in my previous paper that all other known double stars of spectral type B have $P > 1^{\mathrm{d}}.3$. To test this I have made a new diagram[3] showing the values of K plotted against log (10.P)

[1] H. Ludendorff, *Sitzungsberichte der Preussischen Akademie der Wissenschaften, Physikalisch-mathematische Klasse*, **9**, 49, 1924.

[2] J. Schilt, *Publications of the Astronomical Society of the Pacific*, **39**, 160, 1927; A. H. Joy, *Astrophysical Journal*, **64**, 287, 1926; Struve and Hujer, *Astrophysical Journal*, **65**, 314, 1927.

[3] In my former paper on this subject (*Monthly Notices of the Royal Astronomical Society*, **86**, 63, 1925) the two stars μ' Scorpii and V Puppis were omitted because the orbits had been derived from objective-prism plates and the value of K_1 was not known. In the present diagram they are included, K_1 having been estimated from the value of (K_1+K_2). I am indebted to Dr. J. A. Pearce for having called my attention to this omission.

(Fig. 2). The boundary line on the left side, corresponding to the short periods, is very well marked, and there is not one star on the left side of the perpendicular line at log (10.P) = 1.06 (or $P = 1^{d}.2$). Open circles represent 27 Canis Majoris in the two positions $P =$ 1165 days and $p_2 = 0^{d}.9964$. Both points fall outside the area, but p_2 seems to be more likely than P.

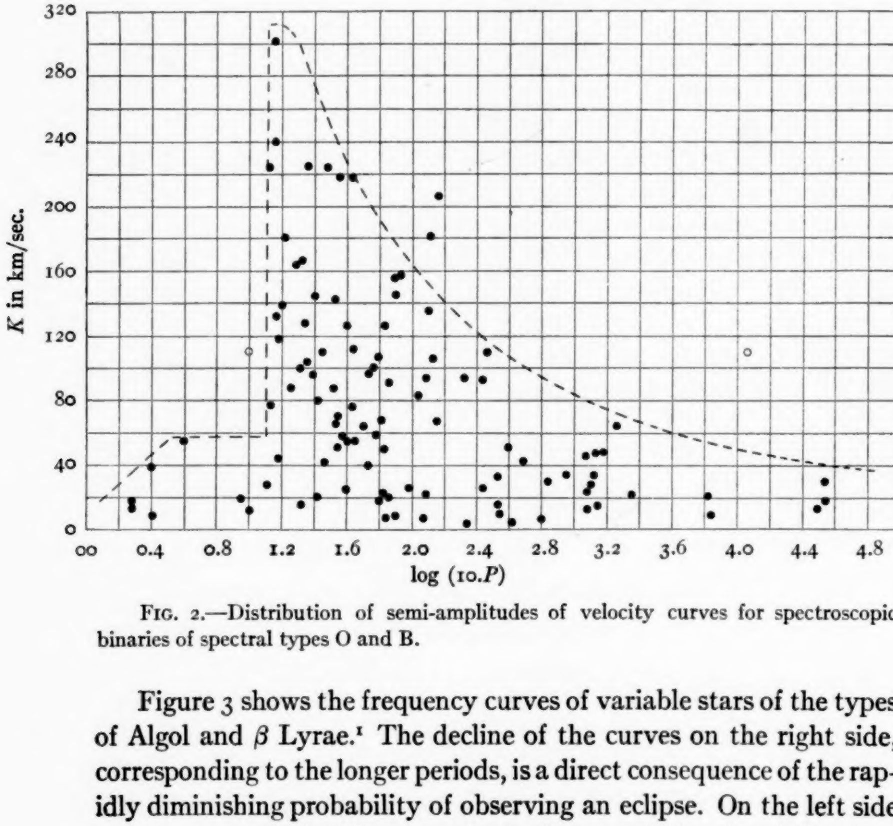

FIG. 2.—Distribution of semi-amplitudes of velocity curves for spectroscopic binaries of spectral types O and B.

Figure 3 shows the frequency curves of variable stars of the types of Algol and β Lyrae.[1] The decline of the curves on the right side, corresponding to the longer periods, is a direct consequence of the rapidly diminishing probability of observing an eclipse. On the left side the curves are limited by the actual existence of binaries of the given spectral type. On the side of the short periods, these branches of the frequency curves correspond to the boundary lines of spectroscopic binaries.[2] For the B stars this line falls at $P = 1^{d}.33$. The only ex-

[1] Source: R. Prager, *Katalog und Ephemeriden veränderlicher Sterne*, 1927.

[2] *Monthly Notices of the Royal Astronomical Society*, **86**, 70, 1925.

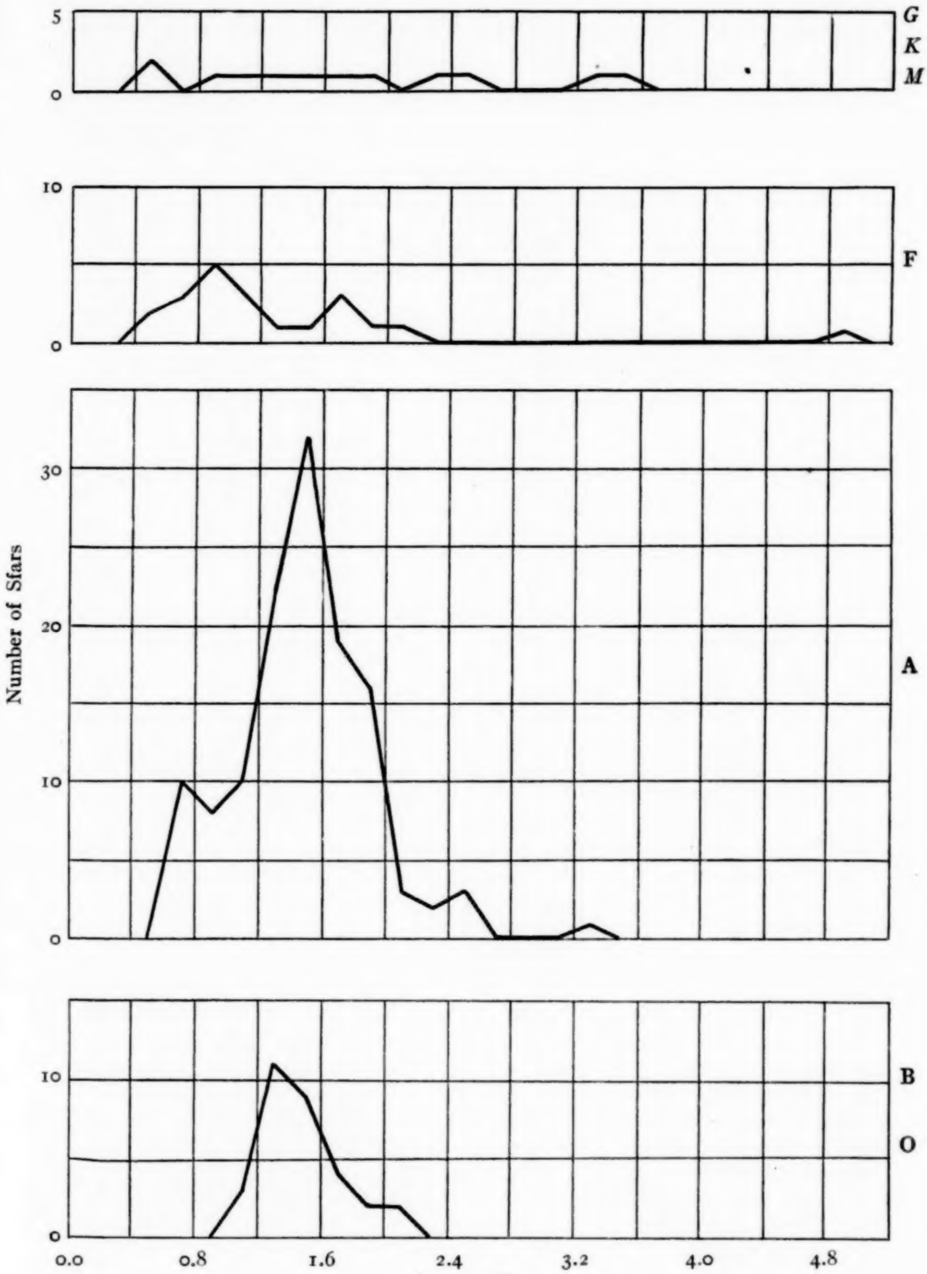

Fig. 3.—Frequency polygons for eclipsing variables

ception is the eclipsing variable EZ Carinae for which P is given as $0^{d}594$. However, upon looking up the literature on this star, I find that E. Hertzsprung, the discoverer, remarks: "Duplication of the period is not unlikely."[1] The spectral type is given as B8. It seemed permissible, therefore, to omit this star.[2]

Again we find that a period of $0^{d}9964$ for 27 Canis Majoris would be unique, although perhaps not impossible.

ECCENTRICITIES OF B-TYPE DOUBLE STARS

From the upper curve in Figure 1 it appears that the orbital elements of 27 Canis Majoris will remain practically the same whether the long or the short period is correct. In each case the eccentricity e is about 0.3. This value, while not extreme, seems rather large for a binary having two components almost in contact. Table II shows that out of 13 spectroscopic binaries with $1^{d}<P<2^{d}$, 12 have $e\leq 0.09$ and only 1 has $e=0.33$. This particular star, however, B.D.+56° 2617, for which P is given as $1^{d}36$, can be represented even better by $P=3^{d}7105$, in which case the eccentricity would also be smaller, of the order of 0.15.[3] Since it is certain that this star should be transferred into column $2^{d}<P<4^{d}$ and $0.10<e<0.20$, it would appear that in this respect, also, 27 Canis Majoris constitutes a unique case. It should be remarked that the values of the eccentricities in Table

[1] *Bulletin of the Astronomical Institutes of the Netherlands*, **3**, 110, 1926.

[2] The frequency curves are interesting also in another way. The left branches recede toward the shorter periods as we proceed from type B to the later types. This means that the smallest late-type stars are smaller than the smallest B stars. The double maximum in type F is undoubtedly due to the division into giants and dwarfs. The curve for G, K, and M is so flat that the discovery of an eclipsing variable of long or of short period appears to be about equally probable.

[3] The period $P=3^{d}7105$ is derived from $p_1=1.3605$ by the relation: $P=p_1/(p_1-1)$. This star was always observed near the meridian, as is evident from the table on p. 271, *Publications of the Dominion Astrophysical Observatory, Victoria*, Vol. **2**. I have tried in the same way to modify the period of the A-type star b Persei, which has an eccentricity larger than that of any other A star of the same length of period. J. B. Cannon (*Publications of the Dominion Observatory, Ottawa*, **1**, 285, 1914) found $P=1^{d}52732$ mean solar days $=1.53005$ sidereal days. Assuming this to be the value of p_1, we find $P=2.8866$ sidereal days. A fairly satisfactory representation of all published observations from Ottawa is obtained with this period. But in this case, the new value of the period does not improve the representation. I am therefore inclined to believe that Cannon's original period is correct. It is quite possible and even probable that other stars will also yield to alternative periods derived in a similar manner.

II, for periods shorter than one day, have been placed in parentheses. These stars are not believed to be real double stars. They are members of the class designated as β Canis Majoris stars and are probably related to the Cepheids.

TABLE II

DISTRIBUTION OF ECCENTRICITIES AMONG SPECTROSCOPIC BINARIES OF TYPES O AND B

Period	0.0–0.5	0.5–1.0	1.0–2.0	2.0–4.0	4.0–6.0	6.0–10.0	10.0–15.0	15.0–25.0	25.0–40.0	40.0–60.0	60–100	100–500	500–1000	1000–
0.00–0.05 . . .	(3)	0	7	13	3	3	4	0	1	0	0	1	1	0
.05– .10 . . .	(1)	0	5	6	3	1	1	0	0	1	0	1	0	0
.10– .20 . . .	0	(1)	0	2	3	3	3	0	0	0	1	2	1	0
.20– .30 . . .	0	0	0	1	0	5	1	2	0	1	1	1	0	2
.30– .40 . . .	0	0	(1)	0	0	2	0	0	1	0	1	1	0	2
.40– .50 . . .	0	0	0	0	0	0	0	0	1	0	0	2	0	1
.50– .60 . . .	0	0	0	0	0	1	0	0	1	0	0	2	0	0
.60– .70 . . .	0	0	0	0	0	0	0	0	0	0	0	0	0	0
.70– .80 . . .	0	0	0	0	0	0	0	0	3	0	0	0	0	0
.80–0.90 . . .	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.90–1.00 . . .	0	0	0	0	0	0	0	0	0	0	0	0	0	0

GENERAL CONCLUSIONS

From the material thus far at hand I do not find it possible to decide in favor of either period. The large mass resulting from P is a strong argument in favor of the short period. On the other hand, the absence of periods shorter than $1^{d}3$ among spectroscopic binaries of types O and B, and, further, the absence of large eccentricities with short periods, would make a period of $0^{d}9964$ appear suspicious. The appearance of the lines, while not giving any conclusive evidence, is also rather against the short period. The same is true of the check observations taken in different hour-angles.

I believe the best course will be to consider either period as possible until the question can be decided by means of observations covering a sufficiently long interval of time during the same night. Such observations could best be made by an observatory in the Southern Hemisphere.

YERKES OBSERVATORY
UNIVERSITY OF CHICAGO
August 6, 1927

ON THE RELATIONS BETWEEN PERIOD, LUMINOSITY, AND SPECTRUM AMONG CEPHEIDS[1]

By HENRY NORRIS RUSSELL[2]

ABSTRACT

If it is assumed that the median total luminosity of periodic variable stars follows the observed mass-luminosity law, and that the variation of any given type arises from a dynamical oscillation of some definite sort, it follows that the existence of a *relation between period and luminosity* demands one between *period and spectral class*, and vice versa. For the *normal Cepheids*, the observed *change in mean spectrum* with *period* is in *excellent agreement* with that computed from the *period-luminosity curve.* If *variability of long period* arises from a dynamical oscillation, the *relation* between *period* and *density* must be roughly the *same* as for the Cepheids. For the *cluster variables*, the *period* corresponding to a given density *must be shorter* than for the Cepheids, and the properties of the dynamical system must be rather peculiar.

Periodic variables decidedly fainter in absolute magnitude than the period-luminosity curve indicates should be *much redder* than Cepheids of the same period, if they owe their variability to *dynamical oscillations.* No cases of this sort are known among the galactic stars.

The existence of a very close correlation between mass and absolute magnitude is an established observational fact, the only exceptions being white dwarfs like the companion of Sirius. It is equally well established that, for stars of the same mass, the effective temperature (or spectral class) has a quite subordinate influence on the brightness.

We may therefore accept as a fact, independent of any theory, a relation of the form

$$L=F(M)T^{m}\ , \tag{1}$$

where L is the total radiation, M the mass, and T the effective temperature of the surface. The form of the function $F(M)$ over the whole observable range is close to that given in Eddington's theory.[3] For small luminosities it varies nearly as M^4, and for great luminosities about as M^2. As for the exponent m, the observational evidence

[1] *Contributions from the Mount Wilson Observatory, Carnegie Institution of Washington*, No. 339.

[2] Research Associate of the Mount Wilson Observatory, Carnegie Institution of Washington.

[3] *Monthly Notices, R.A.S.*, **84**, 310, 1924.

shows only that it is small. Eddington makes it $+4/5$. We may, if we wish, write (1) in the form

$$L = M^n T^m f(M) \; , \tag{2}$$

where, by suitable choice of n, $f(M)$ may be made nearly constant over a range of a few magnitudes in brightness. But we have always

$$L = 4\pi\sigma r^2 T^4 \qquad M = \tfrac{4}{3}\pi\rho r^3 \; ,$$

where σ is Stefan's constant, r the star's radius, and ρ its mean density. Eliminating M and r from (2) we find

$$L^{\frac{3}{2}n-1} f(M) = \text{Const.} \times \rho^{-n} T^{6n-m} \tag{3}$$

For the brighter stars the best value of n is about 2. This gives, according to Eddington's data, the following relative values of $f(M)$:

Abs. Mag......	+1	0	−1	−2	−3	−4	−5	−6
$f(M)$..........	0.64	0.83	1.00	1.13	1.16	1.09	0.97	0.82

Introducing $n = 2$ into (3), we have

$$L = \text{Const.}\; T^{6-\frac{1}{2}m} \rho^{-1} f^{-\frac{1}{2}} \; . \tag{4}$$

This equation is a transformation of that of the curve of mass and luminosity. It is applicable to any star whatever, provided that it is remembered that for absolutely faint stars, or for bright stars (if they exist) which do not follow Eddington's mass-luminosity curve, the values of f may differ seriously from unity. The great importance of the effective temperature in determining the brightness, and the relative insignificance of the imperfectly known constant m and of the correction term f are noteworthy.

We may now apply this equation to the case of Cepheid variables. There is a great deal of evidence that the immediate cause of the variation of all those stars whose variability is "punctually" periodic (as Miss Clerke put it) and not explicable by eclipses is a periodic variation in the effective temperature of the visible surface, ac-

companied perhaps by changes in size or shape, and that behind these changes there is some sort of dynamical oscillation within the system, taking place under gravity. The volume pulsations considered by Eddington and the changes of figure connected with rotational instability and fission considered by Jeans are examples of such oscillations. For any such dynamical oscillation of definite type, the period P and the mean density ρ should be related by an equation of the type

$$\rho = P^{-2} q(P) \; , \tag{5}$$

where q is a slowly varying function.

For similar systems, differing in density but not in physical constitution, q should be strictly constant. In an actual case, physical changes, such as that of the ratio of the specific heats in a pulsating star, or of the degree of internal condensation in a dividing mass, might cause it to vary gradually. For oscillations of different dynamical types, the values of q will be different; but it is well known that, except for higher harmonics, they are usually of the same order of magnitude.

There are, indeed, systems for which the relation between P and ρ is less simple, and depends on other parameters. For example, in an eclipsing pair it involves the radii of the two stars, in terms of that of the orbit. In such systems, however, one would expect that the form of the light-curve would depend upon these additional parameters, as it does for eclipsing variables. For the Cepheids, however, Hertzsprung[1] has given evidence that the remarkable differences in the forms of different light-curves are very closely correlated with the period, stars of the same period having very similar curves; and this is good prima facie evidence that no additional parameters are of fundamental importance, and thus equation (5) is applicable.

The radiation from an oscillating system will vary, and its mean value will presumably differ more or less from that which it would emit if the oscillation should be reduced to zero; but this difference may, in general, be expected to be of the second order with respect to the amplitude of the oscillation. It appears therefore safe to assume that the range of values of L during the oscillation will include

[1] *Bulletin of the Astronomical Institutes of the Netherlands*, 3, 115 (No. 96), 1926.

the equilibrium value. Upon this assumption we may combine (4) and (5), introducing at the same time $m=0.8$, which is abundantly accurate for our purpose, and write

$$L=\text{Const.}\times\frac{T^{5.6}P^{2}}{qf^{\frac{1}{3}}}\,. \tag{6}$$

With proper values of q and f, this equation should hold true for all oscillating systems.

It may next be remarked that there is nothing in these relations—or, indeed, in any relations derivable from the general nature of a dynamical oscillation—to give a period-luminosity curve. No general relation can pick out one distinctive value among the infinity of possible solutions of the dynamical problem for a given mass; such a relation must be based in some way upon the properties of atoms. The relation of mass and luminosity is a case in point; but, though it determines the radiation from a star of given mass and radius, it puts no restriction at all upon the radius of a star of given mass. The existence of a specific relation between period and luminosity, and hence between density and mass, indicates that the atomic (or quantum) properties of matter must again enter the physical problem. How and why they do so is not yet understood; but it is an immediate consequence of equation (6) that, if such a correlation between period and luminosity exists, there must also be an even closer correlation between period and effective temperature. Either one implies and demands the other.

This statement is not new; it is contained implicitly in the equations of Shapley[1] and the theoretical work of Eddington,[2] and Jeans[3] has recently shown how a period-luminosity curve follows, on his hypothesis, from the assumption of constant temperature. This assumption, however, really begs the question, from the standpoint of theory. We can at present give no definite reason why a rotating, or pulsating, star of given mass should find itself in a critical condition for some definite value of the mean density, although we may surmise that changes in ionization have something to do with the mat-

[1] *Mt. Wilson Contr.*, No. 154; *Astrophysical Journal*, **49**, 24, 1919.

[2] *Monthly Notices, R.A.S.*, **79**, 2, 1918.

[3] *Ibid.*, **85**, 808, 1925

ter.[1] From this point, therefore, our procedure must be empirical. We take one of the two relations between period, luminosity, and effective temperature from observation, deduce what the other should be, and compare this with the facts.

According to Shapley's latest period-luminosity curve[2] the visual absolute magnitude of a Cepheid increases from −1.30 to −3.13, for a change in log P from 0.5 to 1.5. The corresponding change in visual magnitude is probably about −2.2, and in bolometric magnitude about −2.5, so that approximately $L \propto P$. It follows from (6) that $\frac{T^{5.6}P}{qf^{\frac{1}{2}}}=\text{Const.}$ If we ignore the changes in f, which are probably quite negligible, and in q, which are likely to be small, we will have $T \propto P^{-0.18}$.

The observed datum is not the effective temperature but the spectral class. For the relation between them we may well adopt the values for giant stars deduced by the writer from various sources,[3] as follows:

Sp.	A0	A5	F0	F5	G0	G5	K0
T	11,000°	8600°	7400°	6500°	5600°	4700°	4200°
Comp.	10,500	8900	7600	6500	5600	4700	4000

The computed values are derived from the formula

$$\log T = 4.02 - 0.14\ (\text{Sp} - \text{A0})\ ,$$

which is substantially identical with one used by Eddington,[4] F0 being taken as 1.0, etc. This is evidently a sufficient approximation for the present purpose. According to it we should expect to find the spectral class advancing by 0.18/0.14 or 1.3 divisions for an increase of 1.0 in log P. It is well to note that the Cepheids actually show an advance in spectral type of just about this amount.

Adams and Joy[5] have determined the spectral classes of more

[1] Cf. Eddington, *The Internal Constitution of the Stars* (Cambridge, 1926), pp. 200–205; Russell, Dugan, Stewart, *Astronomy* (Boston, 1927), **2**, 921.

[2] *Harvard Circular*, No. 280, 1925.

[3] *Op. cit.*, p. 753. [4] *Op. cit.*, p. 181.

[5] *Mt. Wilson Communications*, No. 100; *Proceedings of the National Academy of Sciences*, **13**, 391, 1927.

than fifty Cepheids, allowing for the variation during the period, and reducing to the median value for each star. A freehand curve drawn to represent these results gives the classes F4, G2, and G6 for log P $=0.5$, 1.0, and 1.5, respectively. This close agreement encourages a more detailed test. By starting with Shapley's period-luminosity curve for photographic absolute magnitude A, the bolometric absolute magnitude, B, may be derived by applying E. S. King's color indices[1] and Eddington's corrections from visual to bolometric magnitude.[2] We have then from (6)

$$5.6 \log T = \text{Const.} + \log L - 2 \log P$$

or

$$\log T = 3.85 - \tfrac{1}{14} B - \tfrac{5}{14} \log P \,, \tag{7}$$

where the numerical constant has been determined so as to agree with the general mean of the spectroscopic data.

We thus find:

log P	A	B	log T	Spectrum
0.0	−0.51	−0.82	3.91	A8
0.4	1.14	1.56	3.82	F4
0.8	1.78	2.48	3.74	G0
1.2	2.48	3.6	3.68	G4
1.6	3.37	4.9	3.63	G8
2.0	−4.81	−6.5	3.60	K0

The observed spectra and those derived from equation (7) for the variables observed by Adams and Joy which have periods between one day and one hundred and thirty days are given in Table I and plotted in Fig. 1. Stars for which the variation is not of the normal Cepheid type are marked by asterisks, and cases in which but a single spectrogram has been obtained are marked by daggers.

The differences between the observed and computed spectra, which are given in tenths of a class in the last column, are usually very small. They are considerable for several of the stars at the end of the list, which, however, have not the typical Cepheid variation, but belong to the RV Tauri class, or are still more irregular. Of the

[1] *Harvard Annals*, **85**, 61 (No. 3), 1923.

[2] *Op. cit.*, p. 138.

stars of shorter period, RW Aquilae has a spectrum with hazy lines, very unlike a normal Cepheid, and RY Boötis shows no varia-

TABLE I

OBSERVED AND COMPUTED SPECTRA OF CEPHEIDS

Desig.	log P	Obs. Sp.	Comp. Sp.	O−C	Desig.	log P	Obs. Sp.	Comp. Sp.	O−C
SW Tau	0.20	F3†	F1	+2	SY Aur	1.00	G0	G2	−2
SU Cas	.28	F4	F3	+1	ζ Gem	1.00	G1	G2	−1
TU Cas	.33	F4	F4	0	Z Lac	1.04	G3	G2	+1
SZ Tau	.50	F7	F6	+1	VX Per	1.04	F7†	G3	−6
RT Aur	.57	F6	F6	0	SV Per	1.05	G2	G3	−1
SU Cyg	.58	F6	F7	−1	RX Aur	1.06	F8	G3	−5
Y Aur	.59	F9	F7	+2	SZ Cas	1.13	G1	G4	−3
α U Mi	.60	F7	F7	0	TT Aql	1.14	G2	G4	−2
ST Tau	.61	F8†	F7	+1	W Ser	1.15	G4	G4	0
T Vul	.64	G0	F8	+2	RW Cas	1.17	G5†	G4	+1
VZ Cyg	.69	G2	F8	+4	SZ Cyg	1.18	G6	G4	+2
V Lac	.70	F7†	F9	−2	SV Mon	1.18	G4	G4	0
AP Sgr	.71	G2	F9	+3	X Cyg	1.21	G4	G4	0
δ Cep	.72	F8	F9	−1	RW Cam	1.21	G4	G4	0
X Lac	.73	F9	F9	0	CD Cyg	1.23	G0†	G5	−5
Y Sgr	.76	G0	G0	0	SZ Aql	1.23	G3	G5	−2
RV Sco	.78	G5†	G0	+5	Y Oph	1.23	G3	G5	−2
X Vul	.80	G3	G0	+3	W Vir	1.23	G4	G5	−1
RR Lac	.81	G3	G0	+3	RS Cet	1.24	G0†	G5	−5
XX Sgr	.81	G0†	G0	0	RU Sct	1.29	G5†	G5	0
U Sgr	.83	F9	G0	−1	RY Sco	1.31	G5†	G5	0
U Aql	.84	F9	G0	−1	WZ Sgr	1.34	G8	G6	+2
X Sgr	.84	F7	G0	−3	X Pup	1.41	G7	G6	+1
η Aql	.85	F9	G0	−1	T Mon	1.43	G6	G7	−1
RS Ori	.88	G0	G1	−1	RS Pup	1.62	G4	G8	−3
W Sgr	.88	G0	G1	−1	SV Vul	1.65	G5	G8	−4
W Gem	.90	G1	G1	0	SS Gem*	1.65	G1†	G8	(−7)
RX Gem	.90	G0	G1	−1	UU Her*	1.66	G0	G8	(−8)
RW Aql	.90	F3n	G1	(−8)	S Vul*	1.82	G9	G9	(0)
U Vul	.90	G1	G1	0	V Vul*	1.88	K3	G9	(+4)
S Sge	.92	G0	G1	−1	R Sge*	1.89	G7	G9	(−2)
RY Boo	.95	F3	G1	(−8)	SX Her*	2.01	G3e	K0	(−7)
VZ Sgr	0.98	G0	G2	−2	RX Cep*	2.11	G7	K0	(−3)

* Not a normal Cepheid.
† Only one observation.

tions in radial velocity. The photometric data are poor in both cases. It appears justifiable, therefore, to exclude these stars from further discussion. The residuals for the excluded stars are placed in paren-

theses in the table. It is worthy of note that all but one are negative. Their average value is −0.4 class and the greatest is −0.8.

For the remaining fifty-seven stars, the average difference between the observed and computed spectra is only ±0.16 of a class. The eleven stars for which there is only one spectroscopic observation give ±0.25, and the forty-six others ±0.14.

The part of the discordance which arises from errors in the estimates of spectral class is small. For the forty-five stars for which

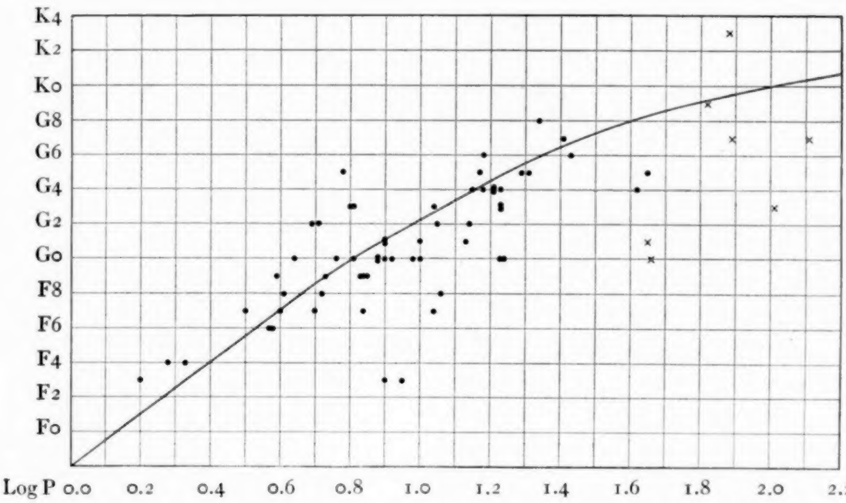

FIG. 1.—Relation between spectrum and period among Cepheid variables. The curve is that derived theoretically in this paper from Shapley's period-luminosity curve.

more than one plate was obtained, the average deviation of a single estimate (corrected for periodic variation of spectrum) from the mean is ±0.094 class. By allowing for the fact that, on the average, a little more than three observations go to a mean, the actual average error of one estimate comes out at ±0.11 class, and that of an average mean value ±0.06 class. Correcting for this, we find the actual divergence from the computed curve to be ±0.13 class for the forty-five stars, corresponding to a probable error of distribution of ±0.11 class, or ±0.0154 in log T, which again corresponds to ±3.5 per cent in the temperature.

It may be concluded from this that the relation between period and spectral class among the typical Cepheids is very closely that which would exist if the stars were all similar dynamical systems, with densities inversely as the squares of their periods, and if their median magnitudes agreed exactly with Shapley's period-luminosity curve. If it is assumed that the quantities q and f in equation (6) are strictly constant, but that the deviations between the observed and computed spectral classes (and temperatures) are real, the mean deviation of ± 0.13 class corresponds to one $0.14\times5.6\times2.5$, or 1.96 times as great in the absolute magnitude: that is, to ± 0.25 mag.

The observed discordance is ± 0.23 mag.,[1] but includes, however, the effect of several other factors. The greatest observed discordance for normal Cepheids, 0.6 class, corresponds to 1.2 mag. For the stars whose variation is abnormal, the largest discordance is -0.8 class, corresponding to -1.6 mag. (if the assumptions are to be trusted here, which is doubtful). It is noteworthy that all the considerable discordances indicate a higher temperature, and hence a brighter absolute magnitude than the normal curve. No observed discordance, for either a normal or an exceptional star, suggests that it is more than a magnitude fainter than the period-luminosity curve indicates. A small systematic error appears in the residuals, however, as shown by the following means:

log P	<0.6	0.6–0.8	0.8–1.0	1.0–1.2	1.2–1.3	>1.3
No. stars	7	10	14	12	8	6
Mean log P	0.44	0.69	0.87	1.09	1.23	1.46
Mean residual	+0.07	+0.12	−0.04	−0.13	−0.20	−0.08

These indicate a change in the residuals from about $+0.1$ to -0.1 class as log P increases by 1.0, which suggests that the factors q and f in (6) are not really constant. The latter is not likely to vary much; if the change is thrown on the former, it would appear that log q must decrease by about 0.16 as log P increases from 0.5 to 1.5, which, by (5), means that the density actually varies over this range, not as P^{-2}, but as $P^{-2.16}$.

Variation in the physical conditions might well account for such

[1] *Harvard Circular*, No. 280, p. 4.

a change. The whole discordance could also be removed by the assumption that the actual difference in absolute magnitude between Cepheids of periods three and thirty days was greater by 0.4 mag. than Shapley's curve indicates; but this is beyond the bounds set by the observations.

This analysis affords strong evidence that a dynamical process, of a single definite type, lies behind the variation of the normal Cepheids. It does not, however, tell us anything more about the process—the hypotheses of pulsation, incipient fission, and probably many others would all meet this condition. Nor does it bear upon the vexed question of the zero-point of the period-luminosity curve. It does, however, afford strong evidence in favor of the existence of such a curve, and the close agreement of the luminosity of most Cepheids with it. It is worthy of special attention that all the considerable disagreements in spectrum are on the whiter side of the curve, indicating that the stars in question, if indeed they are similar dynamical systems, are brighter than the normal for their period. Stars which are fainter than indicated by the period-luminosity curve should be redder than normal, and no evidence of any serious deviation of this kind has been found for stars having periods from one to a hundred days. One or two variables of spectrum Me, which are obviously of the long-period type, though with periods slightly less than one hundred days, are not considered here.

Accepting this as an observational fact, we find it of interest to inquire under what other conditions a dynamically oscillating system could give rise to a variable star considerably fainter in absolute magnitude, but having a period and a surface temperature within the observed range. This can happen only if q or f is large compared with its values for the Cepheids.

Now, for stars which follow the mass-luminosity law, f is nearly constant for the brighter absolute magnitudes and decreases for the fainter ones, Eddington's data giving, on the same scale as above:

Abs. mag.	0	2	4	6	8	10	12
f	0.83	0.47	0.21	0.084	0.031	0.012	0.0045

If a variable is to be faint (for a given density and temperature), it must therefore emit very much more light than the mass-luminosity

law allows. This seems at first paradoxical, but it is a simple consequence of the transformation of equation (2) into (4). No such stars are known, and there appears to be no way of escape in this direction. A larger value of q, by equation (5), signifies that the type of oscillation is such that the period is considerably longer for the same density than in the case of the typical Cepheids. This again seems improbable. For the pulsation of a spherical body, for example, the higher harmonics have shorter periods than the fundamental; and, in the case of rotation, there is no approach to instability for angular velocities much smaller than those which start fission. Periodic changes which depend on alterations of other physical conditions, however, may have far longer periods than those of gravitational oscillations; for example, the sun-spot period is of the order of fifty thousand times the length of a gravitational pulsation of the sun.

It is of interest to apply equation (7) to the periodic variables of other types. Take first the long-period variables. We have no accurate information regarding their bolometric magnitudes, but know that they are very bright. The visual absolute magnitude at maximum lies not far from 0 and the heat index is about four magnitudes. Taking a period of three hundred and thirty days (that of *o* Ceti), we find for $B=-4$, $T=1750^\circ$; and for $B=-5$, $T=2050^\circ$. The latter fits very well with Joy's estimates[1] of a temperature of 2300° at maximum and 1800° at minimum, thus indicating that, if long-period variation is due to a dynamical oscillation, the relation between period and density, in its case, may be much the same as for the Cepheids—which is far from saying that the oscillation is of the same type. The well-known advance of spectral type with increasing period among the long-period variables suggests the existence of a period-luminosity relation among them; but the observational data are not sufficient to test the matter at present.

The case of the variables of the cluster type is puzzling. Adams and Joy have determined the spectral class for thirty stars with periods less than a day. All but three of them are of classes between A4 and F5. The outstanding cases are TX Scorpii, $P=0^{\text{d}}.94$, Sp. A2n; SU Aurigae, $0^{\text{d}}.47$, G2; and RT Scuti, $0^{\text{d}}.50$, M2. The photometric

[1] *Mount Wilson Contr.*, No. 311; *Astrophysical Journal*, **63**, 281, 1926.

data for all three stars are very weak, and it is doubtful whether any one of them is a normal variable of short period. Miss H. B. Sawyer[1] finds no evidence of variation of TX Scorpii on two hundred photographs. Pending further photometric observations, they may be removed from this list.

The remaining twenty-seven stars show no correlation between spectrum and period. The means for successive groups in order of period are

log P..............	9.52	9.66	9.70	9.78
Sp................	A7	A8	F0	A8
Stars..............	7	7	7	6

while the individual spectra are distributed as follows:

A4	A5	A6	A7	A8	A9	F0	F1	F2	F4	F5
1	4	1	7	1	3	3	2	3	1	1

and give an average deviation from the general mean of ± 0.26 class, very little of which can be accounted for by the errors of observation.

For variables of the cluster type Shapley[2] finds a mean visual magnitude of -0.34, corresponding to a bolometric magnitude of -0.5. On the assumption that the constant q is the same for these stars as for the ordinary Cepheids, equation (7) then gives

log P......	9.80	9.60	9.40
log T......	3.96	4.03	4.10
Spectrum ..	A4	A0	(B8)

For the mean value of log P, 9.66, the computed value of log T is 4.01 and the spectrum A1, while the observed spectrum A9 corresponds to log $T = 3.89$. Not a single one of the stars is as white as the formula indicates, and the difference in period between the first and last of the groups given above should correspond to a difference in spectrum of fully 0.5 classes, while the observed difference is zero.

It appears, then, that there must be some real difference between the dynamical processes which are behind variations of the Cepheid type and of the cluster type. (There is, of course, abundant

[1] *Harvard Bulletin*, No. 847, 10, 1927.

[2] *Mount Wilson Contr.*, No. 151; *Astrophysical Journal*, **48**, 89, 1918.

evidence of important differences in the statistical properties of stars of the two groups.) If this difference is thrown on the value of q, it must be assumed that this quantity is, in the mean, 4.6 times greater for the Cepheids than for the cluster variables, or that the period corresponding to the same density is 2.1 times longer in the case of the Cepheids. Such a difference is probably not beyond the range of values which might be expected to result from varying physical conditions. The observed range in spectrum among (galactic) cluster variables of the same period would correspond to an average deviation from the mean absolute magnitude of ± 0.5 mag., which is not large, but much greater than the observed dispersion in globular clusters. The substantial identity of mean spectral class for a range of period in at least the ratio 2:1 indicates, by equation (6), that q must be correlated with the density, and least for the stars of shortest period; in other words, that it deviates more from the value holding good for the Cepheids, as the star is more dense, and indeed that q is proportional to the density. Stars of different densities would therefore have the same period of oscillation.

All this seems extraordinary and improbable; but, unless the mass-luminosity relation fails in this case, the similarity of absolute magnitude among the cluster variables indicates that this type of variation is confined to stars of a certain mass, which is still more remarkable. It is evident that a great deal is yet to be learned before we have any real understanding of the physical causes of variation of this sort, or, indeed, of any other real stellar variability.

It is a great pleasure to acknowledge the writer's indebtedness to his colleagues, Adams and Joy, who generously put all their observations at his disposal.

Carnegie Institution of Washington
Mount Wilson Observatory
June 16, 1927

Addendum.—By some mischance, Shapley's discussion of the same problem (*Harvard Circulars* 313, 314, 1927) did not come to the writer's attention until the present paper was in type. The two discussions are based on independent observational data. They proceed along similar lines, and reach entirely accordant conclusions. Each one, however, covers some points not included in the other.

ORBIT OF THE SPECTROSCOPIC BINARY 95 *o* LEONIS

By O. STRUVE and W. W. MORGAN

ABSTRACT

The *orbital elements* of this spectroscopic binary, which has *double lines*, were derived from 35 spectrograms, of which 33 were obtained with the one-prism Bruce spectrograph of the Yerkes Observatory. $P = 6^{d}.6254$; $\gamma = -20.4$ km/sec.; $e = 0.02$; $K_1 = 57.6$ km/sec.; $\omega = 4^{\circ}.1$; $T = 1927$ March 1.115 U.T.; $m_2/m_1 = 0.72$; $(m_1 + m_2) \sin^3 i = 1.8 \odot$; $a_1 \sin i = 5{,}250{,}000$ km.

Variations in the radial velocity of 95 *o* Leonis were discovered at the Mount Wilson Observatory and announced by Dr. W. S. Adams,[1] in 1912. The binary nature of this star was independently discovered at the Yerkes Observatory by Professor S. B. Barrett, who noted that on the first plate taken February 2, 1914, the lines were "widely double." After we learned from the Mount Wilson observers that they did not intend to determine the orbit of the binary it was placed on the regular observing program of the Bruce spectrograph for the spring of 1927. Our orbit is based on 35 spectrograms, of which 2 were taken at Mount Wilson and 4 at Yerkes Observatory in 1914–1917. All of our plates were obtained with the one-prism arrangement, which gives a linear dispersion of 30 A per millimeter at λ 4500.

The position of 95 *o* Leonis for 1900 is α $11^{h}50^{m}.5$, δ $+16^{\circ}12'$. According to the *Henry Draper Catalogue*, its spectral type is A2 and its photometric magnitude is 5.49. The metallic lines are fairly sharp and well defined. Magnesium λ 4481 and iron λ 4045 are best. On good plates from ten to twelve stellar lines were usually measured.

Table I contains the radial velocities used in the computation. The measurements of all Yerkes plates were made by Struve. As a check on these values some of the plates were independently measured by Morgan, but his results are not included in the table, nor were they used in the solution. The velocity of the second component has been measured whenever possible. Its spectral type

[1] *Astrophysical Journal*, **35**, 175, 1912.

seems to be the same as that of the primary, while its lines are appreciably fainter. In view of the relative uncertainty of the meas-

TABLE I

RADIAL VELOCITIES OF 95 o LEONIS

Date U.T.	Observer	Quality	Vel. I	Vel. II	O.−C.
1911 Dec. 21.022	Mt.W.	....	−90.0	....	....
1912 Jan. 25.001	Mt.W.	....	+ 7.0	....	....
1914 Feb. 2.851	F.B.S.	g	−67.2	+105	....
1914 Feb. 25.753	F.S.	f	+42.6	− 66	....
1914 Mar. 16.860	B.F.S.	p	+13.9	....	....
1917 Jan. 1.968	Mk.S.	g	+42.4	− 57	....
1927 Mar. 5.287	σ S.	g	−53.3	+ 86	+ 3.6
1927 Mar. 8.398	σ H.S.	g	+22.8	....	− 1.3
1927 Mar. 9.198	σ H.S.	g	−17.7	....	− 2.6
1927 Mar. 9.262	σ H.S.	g	−18.3	....	+ 0.3
1927 Mar. 9.330	σ M.S.	g	−20.1	....	+ 2.2
1927 Mar. 9.393	M.S.	g	−20.0	....	+ 5.8
1927 Mar. 9.459	M.S.	g	−28.4	....	+ 0.9
1927 Mar. 10.184	M.S.	g	−67.5	+ 82	− 4.8
1927 Mar. 10.397	σ H.S.	f	−73.0	+ 99	− 3.7
1927 Mar. 18.137	σ H.S.	f	−72.6	....	+ 2.6
1927 Mar. 22.205	M.S.	g	− 5.0	....	− 3.3
1927 Mar. 22.385	σ H.S.	g	−12.0	....	− 0.6
1927 Mar. 27.435	M.S.	p	+46.0	....	+ 8.1
1927 Mar. 28.111	M.S.	f	+25.9	− 79	− 3.6
1927 Apr. 6.176	σ H.S.	g	−79.5	+ 75	− 4.6
1927 Apr. 7.110	M.S.	g	−71.9	+ 85	− 4.3
1927 Apr. 7.177	M.S.	f	−70.2	+ 64	− 4.6
1927 Apr. 7.311	M.S.	f	−61.3	+ 72	− 0.3
1927 Apr. 14.097	σ S.	p	−50.8	....	+ 3.8
1927 Apr. 20.250	σ S.	f	−60.3	....	+10.3
1927 Apr. 24.141	M.S.	g	+ 3.2	....	− 5.0
1927 Apr. 30.151	σ H.S.	g	+31.8	−103	− 0.2
1927 May 4.185	σ H.S.	g	−53.1	....	− 8.1
1927 May 13.101	σ H.S.	g	+33.6	−110	− 3.8
1927 May 17.121	M.S.	g	−65.0	+ 49	− 6.1
1927 June 1.125	M.S.	f	+39.4	....	+10.2
1927 June 10.181	σ H.S.	p	−37.7	....	−10.1
1927 June 15.120	σ S.	g	+32.7	....	+ 0.3
1927 June 16.131	M.S.	g	+10.7	....	+ 1.6

The names of the observers and the quality of the plates are designated as follows: Mt.W. = Mount Wilson; B = S. B. Barrett; F = E. B. Frost; Mk = G. S. Monk; M = W. W. Morgan; H = C. Hujer; S = F. R Sullivan; σ = O. Struve; g = good; f = fair; p = poor. The decimal fractions of the day are expressed in Universal Time, except for dates prior to 1925 for which Greenwich Mean Time is given.

ures of this fainter component, we have not taken them into consideration in the computation of the orbit. They have been used, however, for the determination of the mass-ratio.

The period was determined, by trial, from the observations of 1927, and was later adjusted to represent the earlier observations. The preliminary elements were derived by the methods of Lehmann-

TABLE II

ORBITAL ELEMENTS OF 95 *o* LEONIS

Element	Designation	Preliminary	Final	Probable Error
Velocity of system.....	γ	−21.0 km/sec.	−20.4 km/sec.	....
Period...............	P	6ᵈ6254	6ᵈ6254	....
Time of periastron.....	T	J.D. 24941.000	.115 U.T.	± 0.19
Longitude of periastron.	ω	0°0	4°1	±10°
Eccentricity...........	e	0.05	0.02	± 0.01
Semi-amplitude........	K	60.0 km/sec.	57.6 km/sec.	± 0.3
Minimum mass........	$(m_1+m_2)\sin^3 i$	....	1.8☉	....
Mass-ratio............	m_2/m_1	....	0.72	± 0.08
Semi-major axis.......	$a \sin i$	....	5,250,000 km	....

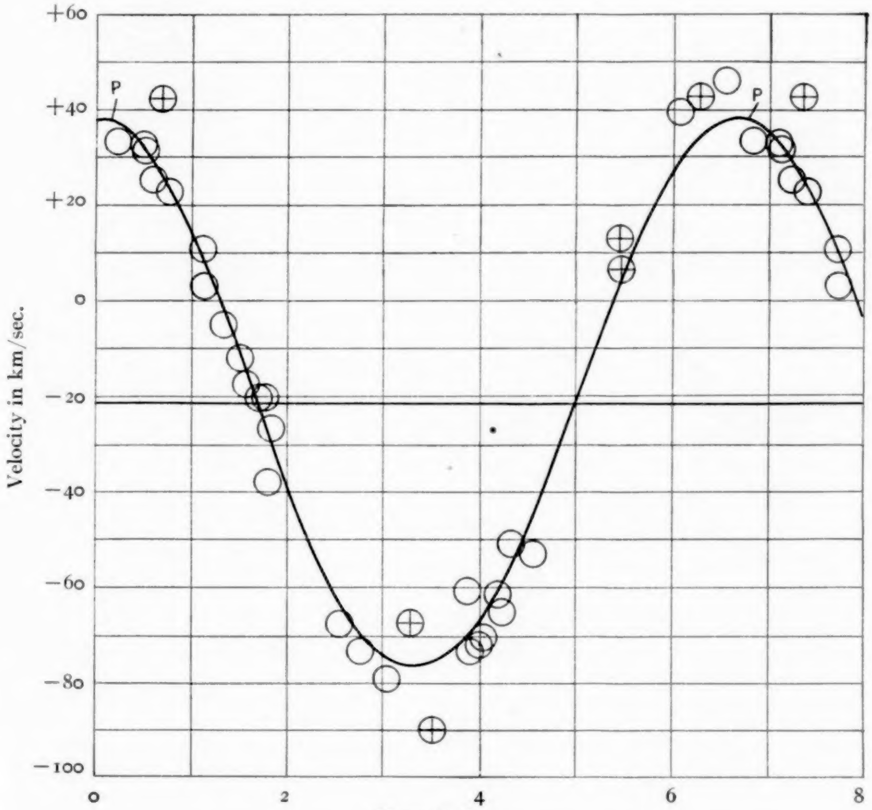

FIG. 1.—Velocity curve of 95 *o* Leonis. Phase zero corresponds to 1927 March 1.000 Universal Time.

Filhés and of Schwarzschild and Laves. A least-squares solution for five unknowns was then made, using only the material of 1927. The period was assumed as constant. A separate observation equation was computed for each radial velocity. These were weighted according to the quality of the plate. Good plates were given weight 1, while fair and poor plates were weighted as 0.6 and 0.3, respectively. Professor Schlesinger's method was closely followed in making the solution. The improvement of the representation is considerable $[p\Delta^2]$ being reduced from 575 to 463. The probable error of one observation of unit weight is ± 2.9 km/sec. The mass-ratio was determined by taking the mean of the values $\frac{(V_1-\gamma)}{(V_2-\gamma)}$. The elements are collected in Table II. Since the final eccentricity is small, the weights of ω and T are low and their probable errors are large. It would perhaps be possible to improve the results by solving a new set of equations for γ, T, and K, but the observations are not regarded as sufficiently accurate to justify this computation.

The velocity curve together with the individual observations is shown in the figure. Zero phase corresponds to J.D. 2424941.000 Universal Time (meaning that Greenwich Civil Time has been used in the decimal fraction of the day, and not Greenwich Mean Time, as is frequently the practice of observers of variable stars). The individual observations of 1927 are represented by open circles. Crosses indicate the earlier observations. The letter P denotes the time of periastron passage.

YERKES OBSERVATORY
July 1, 1927

REVIEWS

Constitution et évolution de l'univers. By A. VERONNET. Paris: Gaston Doin et Cie., 1926. Pp. iv+475. Fr. 20 net.

The problems of cosmogony are of perpetual interest, and in this small volume of 475 pages an astronomer at the University of Strasbourg, A. Veronnet, gives us the picture as he sees it after fifteen years of study upon certain mathematical phases of the subject. The point of view is that of the mathematicians of the nineteenth century, just as was that of Jeans in his *Problems of Cosmogony* about eight years ago. Since the publication of his book, Jeans has advanced to more modern ideas as to the nature of matter and energy, but Veronnet prefers to plod along the old path, and it is a curious picture which he presents to us.

One would suppose that a modern mathematician would feel the need of a few postulates before expounding his views upon the universe in general. Veronnet does not feel this need. His postulates are carefully concealed (perhaps he was not aware of them), and a casual reader is left with the impression that everything is a matter of calculation and therefore quite certain. With a word of caution to the reader that a mathematical discussion, like every other logical discussion, rests upon a variety of assumptions which may or may not be granted, we will proceed to a description of the evolution of the universe as it is seen by our author.

Before the stars were formed all matter existed in the form of atoms and molecules scattered more or less uniformly throughout all space. Each molecule attracted every other molecule, at least those within a certain distance, and the resultant attraction on each molecule was perfectly determined. (The reviewer would remark that these assumptions imply that Newton's law of gravitation is only an approximation; and so fundamental an assumption should be explicitly stated and discussed.) Previous to this, however, "Even matter did not exist. Only the ether, the universal electromagnetic medium, filled space with positive and negative electrons." (Most of us would say that electrons were matter.) The ether and electrons came into existence somehow from some previous state but no conjecture is made as to how, nor why the number of positive and negative electrons are equal in number and equal, though opposite, in charge. The author is content with the remark that nothing is known on

the subject. Gravitation is a feeble residue which remains after the electrons have been combined in the form of atoms. (It should therefore vary inversely as the cube, or some higher power, of the distance.)

The primitive nebulosity was not absolutely uniform—it was more like the clouds in a mackerel sky—and no initial motions of the atoms or molecules are assumed. The stars began to condense in the midst of the denser portions of the nebulosities without any sharp boundaries at first; the space between the stars being cleared gradually as time elapsed. The first fragments to condense were the smaller ones of a few tons or thousands of tons like comets and meteors, the stars being condensations of a second order. Naturally, the temperatures to which the stars rise depend upon the initial masses, and thus stars of all types are to be found from the white to the red. (Veronnet fails, however, to explain why there are no stars a million or more times as massive as the sun.) The star clusters of various types are condensations of the third order—fourth, and higher orders are not mentioned.

The process of evolution has proceeded more rapidly in some regions than in others. In our stellar world (the galaxy?) the evolution of the stars, and *a fortiori* of meteorites and atoms, is completed; but the concentration into star clusters has little more than commenced. The condensation of the clusters is in process and eventually, when they have reached mechanical equilibrium, their diameters will be much smaller than they are at present. On the borders of the galaxy the evolution is more advanced, and outside of it still more. Spiral nebulae are formed by the approach of two clusters and their rotation about each other like double stars. The galaxy is merely a region where the evolution of clusters has not yet made much progress.

As for the planets, they were originally fragments of the primitive nebulosity, but, on account of their remoteness from the central condensation that eventually became the sun, they were deflected from a direct descent into the sun by the attraction of neighboring stars. Their orbits, at first very elongated, have been rounded by the resisting medium. Furthermore, the sun possesses more than the single planetary system with which we are familiar; it possesses many planetary systems. The others, formed later, are very remote and have highly elongated orbits like comets. This total mass exceeds that of the sun by at least ten times.

Other stars possess similar planetary systems more or less remote. Occasionally one of the nearer planets falls into a star which then becomes a nova. From the nova the star passes to the giant stage by virtue of an

envelope of particles which is maintained about it by radiation pressure. Before the infall of the planet it was an ordinary dwarf star.

The heat of the sun is derived from the sun's gravitational potential in accordance with Helmholtz' theory, but it must not be supposed that the sun's radiation has been uniform. Its mean radiation in the past has been fifteen times its present radiation, so that the age of the sun and its attendant system of planets is only one million or so years. (Jeans assumed in his *Problems of Cosmogony* that for the first two hundred million years the radiation of the sun was one-fortieth of its present radiation, the present rate extending back into the past only five million years.) This need not disturb the geologists, for under this intense radiation water would be boiling, and with boiling water the process of erosion would be a thousand times faster than the present rate. As for the radioactive method of determining the age of the earth, "It seems indeed very uncertain to base a calculation of millions of years upon a few milligrams of helium," and anyway it is only a fragment of rock which was analyzed, and as the earth was formed of meteorites, the rock may have been very old when it fell in.

So much for the past! In the future the sun will continue to cool and the earth also, unless it should happen to fall into the sun. The energy of the sun is radiated away and is lost, so that the universe as we know it now is but a transitory phase. The individual stars cool and become dark. The star clusters will collapse into single stars of great mass and brilliancy which will have very long periods of life. As time elapses the centers of radiation become relatively fewer but more massive. Since the universe is infinite it is not necessary to suppose that the process will ever end, for the sweeping-up process can go on forever. The phase of the life of the universe which we see is merely that phase in which the gravitational potential energy is being transformed into radiant energy. There is no question raised as to the phase of the universe which succeeds the radiant phase. The radiated energy simply disappears from consciousness.

Such is the fantastic picture which is presented by Veronnet. It is offered to us with numerous mathematical formulae as to details, but with no evidence whatever of any meditation upon the larger or philosophical aspects of the subject. Apparently he has accepted the traditional point of view in which he was educated and seeks merely to make it more precise as to details. Whether the universe has emerged from some unknown state and is proceeding through the present conditions to some other equally unknown state, or whether the universe remains always in its present state in the sense that atoms and stars, radiant energy and

potential energy, have always existed and always will exist is a postulational matter that cannot be settled by experiment, observation, or mathematical discussion. Consequently, the second law of thermodynamics, however useful it may be as a laboratory rule, cannot be accepted as a philosophical principle on the basis of evidence. If the reviewer's experience in testing people is of any value, most people distinctly dislike the postulate that the universe is proceeding from one unknown to another, and with equal distinctiveness favor the postulate that the universe remains always the same; it is a matter of taste, and *de gustibus non est disputandum*. Nevertheless, there is no point in taking up with the unpopular side with no mention whatever of the existence of the other.

There is no evidence, however, that Veronnet consciously made any choice. He pins his faith to the Helmholtz contraction theory and then makes his philosophy conform to the theory, instead of taking it the other way about by making his theory fit his philosophy.

The idea that the heat of the sun is maintained at the expense of the sun's mass is erroneously ascribed to Perrin (1919) on relativistic grounds. This hypothesis was first advocated by the present reviewer on purely classical grounds in this *Journal* in 1918. It has also been advocated by Nernst (1921) and by Jeans and Eddington (1924). The vast amount of energy which is obtained in this manner is regarded by Veronnet as a fairy-like contribution by the relativists, not to be taken seriously. He is wrong, however, in stating that the gravitational potential energy is the only adequate source of energy that we really know about. The electrostatic potential energy is just as real (in the classical sense) as the gravitational potential energy, and it is vastly greater, the ratio of the electrical forces to the gravitational forces on the electron being 2×10^{39}.

If one admits that the electrons cease to exist when a positive and a negative electron approach as close as the radius of the positive electron, the amount of energy released is almost exactly that given by the theory of relativity. Curiously enough, Veronnet seems willing to admit that atoms have come into existence, but seems unwilling to let them go out of existence. Jeans and Eddington, on the other hand, seem willing to admit that they go out of existence but not willing to admit that they come into existence. In the cosmology of the reviewer the atoms, like all other physical units, come into existence and go out of existence. Their construction makes the skies of night cold and dark; their destruction is the source of the stellar energies. The time scale is not a few million years, as Veronnet would have it, but millions of billions of years. The geologists and biologists will regard an estimate of one million years for

the age of the earth as utterly absurd—and still more absurd will it appear to the student of the dynamics of the galaxy.

Veronnet has heard of the planetesimal hypothesis of the origin of the planetary system, for he says with regard to it:

> One sees none of the necessary mechanical conditions fulfilled here. The sun turns much too slowly to give a sufficient moment of momentum to the planets. It could give them only a part of its moment of momentum while with a mass of merely 1/1000 they possess 30 times as much. It is therefore an absolute impossibility.

This opinion shows that the author is wholly ignorant of the planetesimal hypothesis. It is a pity that he mentioned the matter at all. He could be pardoned if he had never heard of it, but having heard of it and having mentioned it he was under obligations to represent it correctly. As a matter of fact, the planetesimal hypothesis cannot be reduced to a set of mathematical equations, but neither can geology, paleobotany, and paleontology. The time is past when all of the secrets of the universe can be read out of a set of mathematical equations, and Veronnet seems to have devoted himself exclusively to this branch of knowledge. We are tempted to paraphrase the quotation, "A little learning is a dangerous thing," and make it read, "The exclusive use of mathematics is a dangerous thing in cosmology."

The book is printed on paper of poor quality, and the proofreading has not been carefully done.

W. D. MacMillan

Statistical Mechanics with Applications to Physics and Chemistry. By Richard C. Tolman. New York: Chemical Catalog Co., 1926. Pp. 334. $7.00.

The book is one of the "Monograph Series" of the American Chemical Society. Its purpose is to give in a single place a systematic presentation of the theory of statistical mechanics, together with a survey of most of its fields of application in physics and chemistry. Professor Tolman's book should be of great importance to the chemist, and the physicist, also, will find much of interest between its covers.

After a very brief treatment of the canonical equations of Hamilton, the statistical ensembles of Gibbs, Boltzmann, Jeans, and Ehrenfest are considered. The Maxwell-Boltzmann distribution law is next derived and applied to problems involving molecular velocities, energy partition, and specific heats. At this point the quantum theory, in the form expounded by the Wilson-Sommerfeld rule of quantization and Bohr's frequency

postulate, is introduced. The quantum theory is then incorporated into statistical mechanics in the usual manner, though here the author proposes an original method which has many good points. In this method the functional relation connecting energy of molecule with co-ordinates and momenta is changed. The modified theory is used in the derivation of the specific heats of solids and diatomic gases, and the laws of distribution for radiation at thermal equilibrium. The larger portion of the applications of statistical mechanics is made to problems in rate of physical-chemical change, such as the determination of viscosity, thermal conductivity, diffusion, electrical conductivity, rate of chemical reaction, and rate of photo-chemical reaction. Applications are also made to molecular processes.

As is to be expected of a book in physical chemistry, there is a lack of rigor in the mathematics. However this book is superior to most works of this type in its mathematical logic. The physical concepts are exceedingly well emphasized. The purpose of the book—to give in a single place a systematic presentation of the theory of statistical mechanics, together with a survey of most of its fields of application in physics and chemistry—has been fully and carefully carried out.

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